

TOMA DE DECISIONES BAJO INCERTIDUMBRE EN MERCADOS ELÉCTRICOS

CLASE 3: “TOMA DE DECISIONES BAJO INCERTIDUMBRE”

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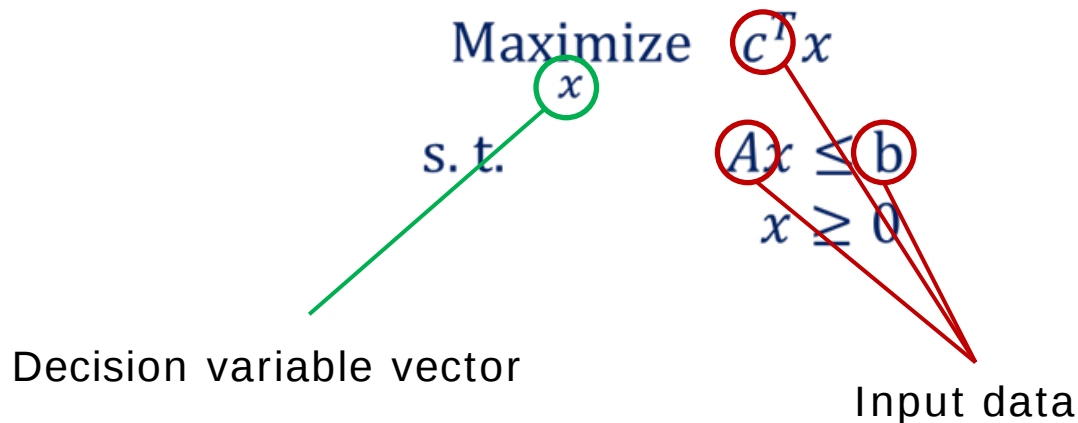
Optimization under Uncertainty

- Decision problems are often formulated as optimization problems
- Consider the following linear optimization problem:

$$\begin{array}{ll}\text{Maximize}_{x} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0\end{array}$$

Optimization under Uncertainty

- Decision problems are often formulated as optimization problems
- Consider the following linear optimization problem:



How to find the best value of “ x ”?

- If A, b and c are perfectly known $\Rightarrow$ deterministic solution algorithms
- Any suggestion?

$$\begin{array}{ll}\text{Maximize} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0\end{array}$$

How to find the best value of “ x ”?

- If A, b and c are perfectly known $\Rightarrow$ deterministic solution algorithms
- Simplex method

$$\begin{array}{ll}\text{Maximize} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0\end{array}$$

How to find the best value of “x”?

- But what if...
 - ✓ A, b and/or c depend on the realization λ_ω of a certain random parameter λ , i.e. $A(\omega) = A(\lambda_\omega), b(\omega) = b(\lambda_\omega)$, and/or $c(\omega) = c(\lambda_\omega)$
 - ✓ Decision vector x needs to be determined before λ is disclosed

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$$\begin{array}{ll}\text{Maximize} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0\end{array}$$

How can we guarantee feasibility?

How to find the best value of “ x ”?

- But what if...

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$$\begin{array}{ll} \underset{x}{\text{Maximize}} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0 \end{array}$$

A family of random variables: $f(x, \omega) = c(\omega)^T x$
Which one is the best?

How to find the best value of “x”?

- But what if...
 - ✓ A, b and/or c depend on the realization λ_ω of a certain random parameter λ , i.e. $A(\omega) = A(\lambda_\omega), b(\omega) = b(\lambda_\omega)$, and/or $c(\omega) = c(\lambda_\omega)$
 - ✓ Decision vector x needs to be determined before λ is disclosed

$$\begin{array}{ll}\text{Maximize} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0\end{array}$$

How can it be reformulated so that it can be processed by deterministic routines?

Optimizing under uncertainty

... encompasses models, methods, and decision theory concepts to deal with feasibility, optimality and solution procedures in optimization problems involving uncertain data.

Stochastic Programming Community Home Page

<http://stoprog.org/>

Decision Making in Electricity Markets

- Uncertainty is present in most decision-making problems faced by electricity market agents
- Examples?
 - ✓ Producers/retailers/consumers: Trading and risk management
 - ✓ TSO/MO: Market-clearing procedures
 - ✓ Increased uncertainty due to the growth of stochastic producers (in particular, wind generation)
 - ✓ Generation and transmission capacity investments

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Solution Approaches



Stochastic programming
with recourse



Chance-constrained
stochastic programming

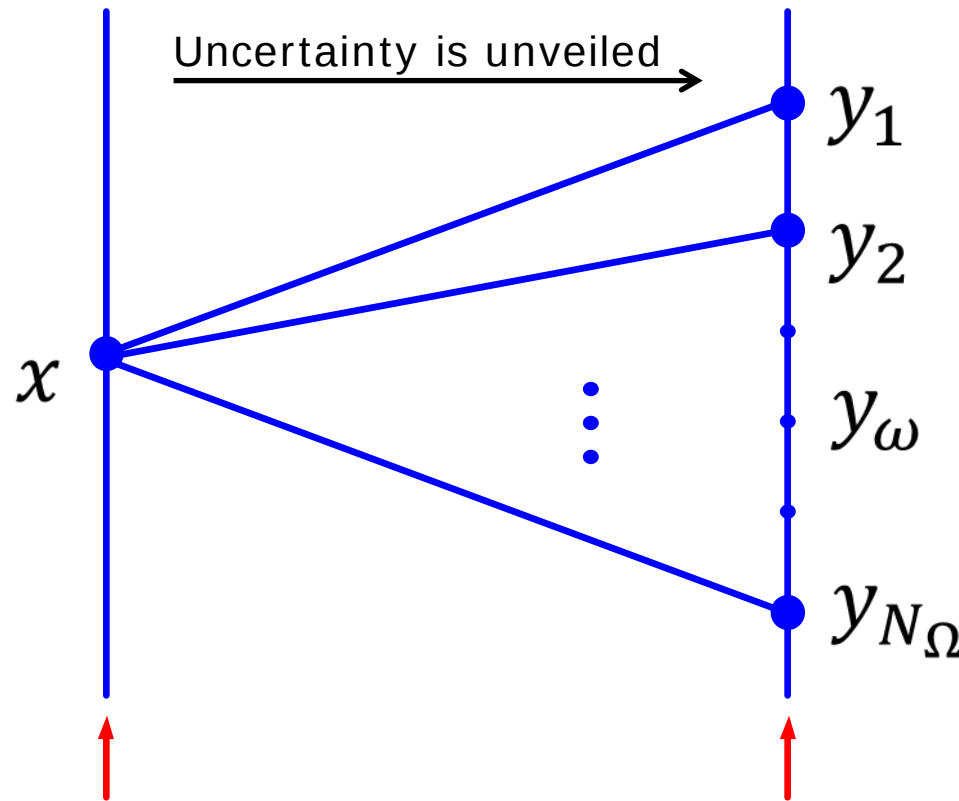


Robust optimization

Stochastic Programming with Recourse

- Two types of decisions:
 - ✓ Here-and-now decisions: made before the uncertainty is disclosed
 - ✓ Wait-and-see or recourse decisions: made once the uncertainty is revealed
 - o They compensate for any bad effect that might have been experienced as a result of the here-and-now decisions

Stochastic Programming with Recourse



Uncertainties:

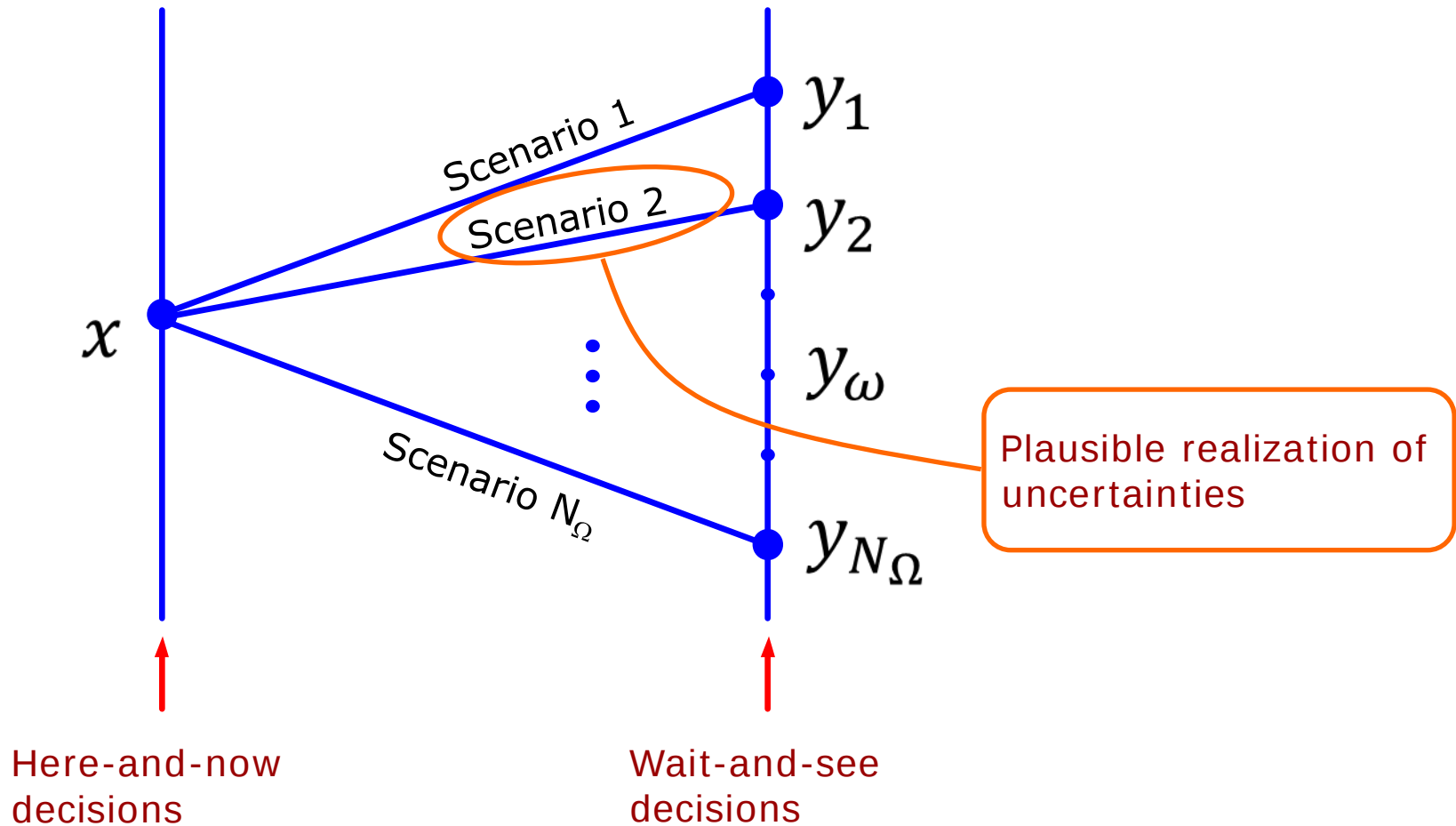
- Wind production
- Equipment failures
- Demand

Scenario tree

Here-and-now
decisions

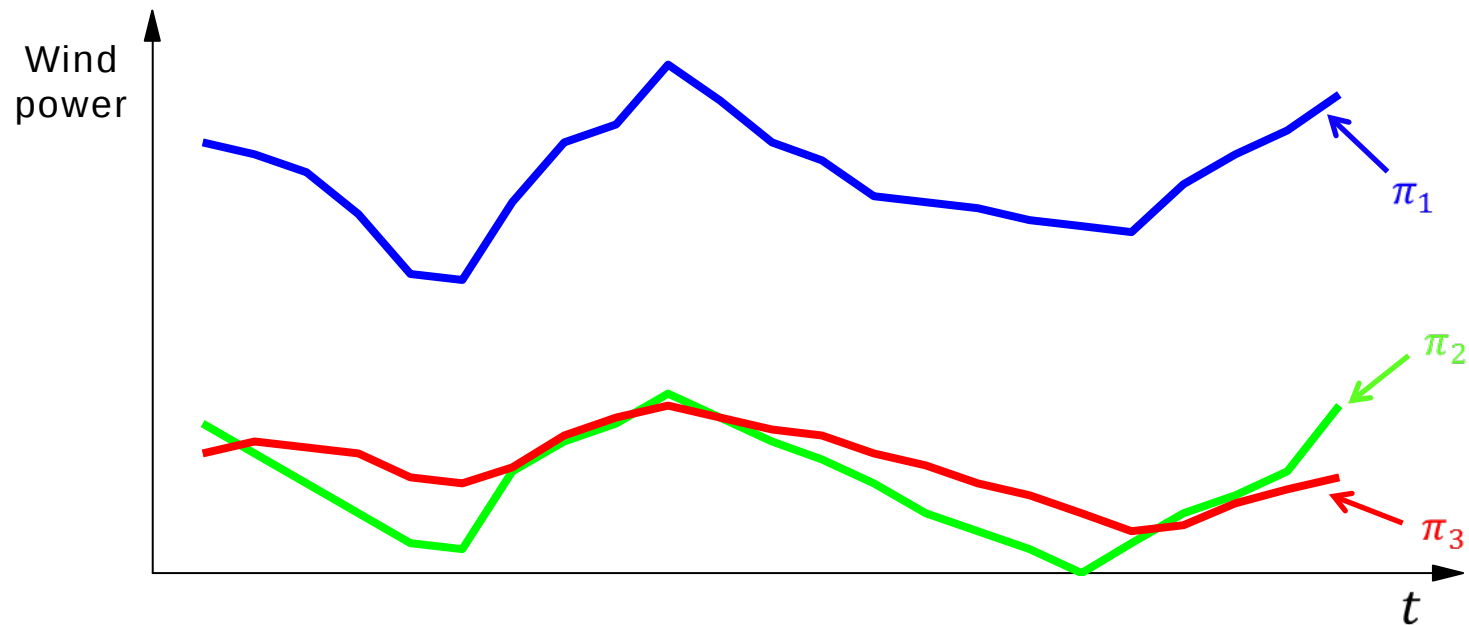
Wait-and-see
decisions

Stochastic Programming with Recourse



Stochastic Programming with Recourse

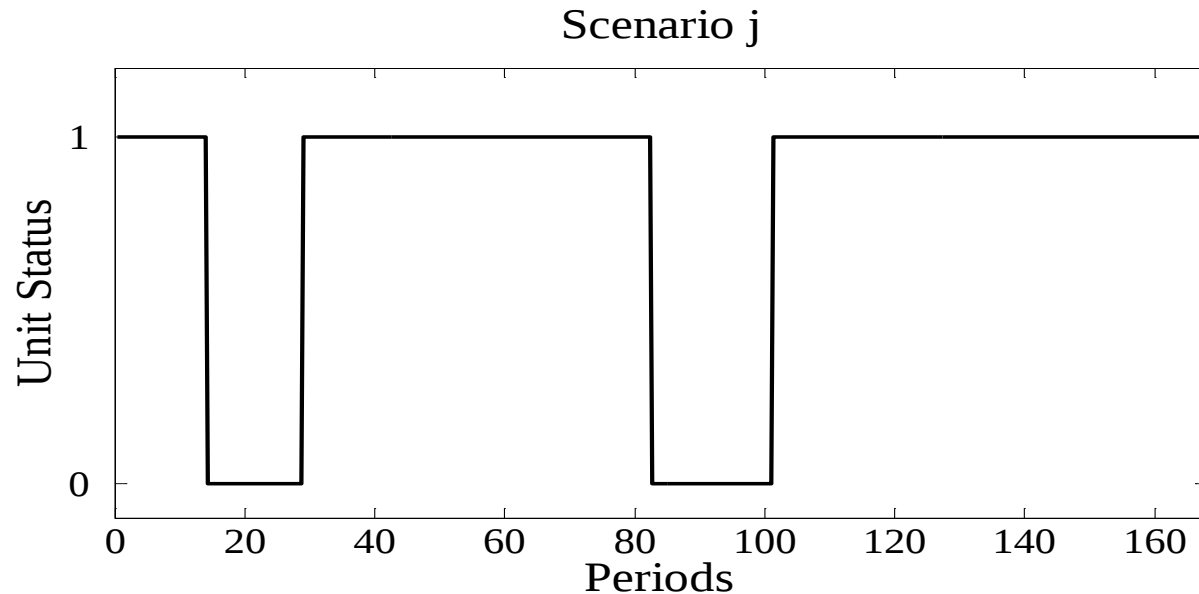
Plausible time trajectories ...



π_ω : Probability of occurrence

Stochastic Programming with Recourse

The probability distributions governing the data are assumed to be known!



$[1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ \dots \ 0 \ 1 \ 1]$

Scenario generation and reduction techniques are imperative!

Stochastic Programming with Recourse

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + E_{\omega}\{Q(x, \omega)\} \\ \text{s.t.} &&& Ax \leq b \\ &&& x \geq 0 \end{aligned}$$

being

$$Q(x, \omega) = \left\{ \underset{y_{\omega}}{\text{Maximize}} \quad q(\omega)^T y_{\omega} \right. \\ \left. \begin{array}{l} \text{s.t.} \\ Wy_{\omega} \leq h_{\omega} - T_{\omega}x \\ y_{\omega} \geq 0 \end{array} \right\}, \forall \omega \in \Omega$$

Stochastic Programming with Recourse

Here-and-now/first-stage decisions

$$\begin{aligned} & \underset{x}{\text{Maximize}} \quad c^T x + E_{\omega}\{Q(x, \omega)\} \\ & \text{s.t.} \quad Ax \leq b \\ & \quad \quad x \geq 0 \end{aligned}$$

being

Wait-and-see/recourse/second-stage decisions

$$\begin{aligned} Q(x, \omega) = \left\{ \underset{y_{\omega}}{\text{Maximize}} \quad q(\omega)^T y_{\omega} \right. \\ \left. \text{s.t.} \quad Wy_{\omega} \leq h_{\omega} - T_{\omega}x \right. \\ \left. \quad \quad y_{\omega} \geq 0 \right\}, \forall \omega \in \Omega \end{aligned}$$

Stochastic Programming with Recourse

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + E_{\omega}\{Q(x, \omega)\} \\ \text{s.t.} &&& Ax \leq b \\ &&& x \geq 0 \end{aligned}$$

being

Recourse problem

$$Q(x, \omega) = \left\{ \underset{y_{\omega}}{\text{Maximize}} \quad q(\omega)^T y_{\omega} \right. \\ \left. \begin{array}{l} \text{s.t.} \quad Wy_{\omega} \leq h_{\omega} - T_{\omega}x \\ y_{\omega} \geq 0 \end{array} \right\}, \forall \omega \in \Omega$$

Stochastic Programming with Recourse

$$\text{Maximize}_x \quad c^T x + E_{\omega}\{Q(x, \omega)\}$$

$$\text{s.t.} \quad \begin{aligned} Ax &\leq b \\ x &\geq 0 \end{aligned}$$

Feasibility is guaranteed for every scenario
(every possible state of nature)

being

$$Q(x, \omega) = \left\{ \begin{aligned} &\text{Maximize}_{y_{\omega}} \quad q(\omega)^T y_{\omega} \\ &\text{s.t.} \quad \boxed{Wy_{\omega} \leq h_{\omega} - T_{\omega}x} \\ &\quad \quad y_{\omega} \geq 0 \end{aligned} \right\}, \forall \omega \in \Omega$$

Stochastic Programming with Recourse

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + \mathbb{E}_\omega \{Q(x, \omega)\} \\ & \text{s.t.} && Ax \leq b \\ & && x \geq 0 \end{aligned}$$

being

Function that takes a random variable and returns a real value

$$Q(x, \omega) = \left\{ \begin{array}{ll} \underset{y_\omega}{\text{Maximize}} & q(\omega)^T y_\omega \\ \text{s.t.} & Wy_\omega \leq h_\omega - T_\omega x \\ & y_\omega \geq 0 \end{array} \right\}, \forall \omega \in \Omega$$

Stochastic Programming with Recourse

$$\begin{array}{ll}
 \text{Maximize}_{x} & c^T x + \boxed{E_{\omega}\{Q(x, \omega)\}} \\
 \text{s.t.} & Ax \leq b \\
 & x \geq 0
 \end{array}$$

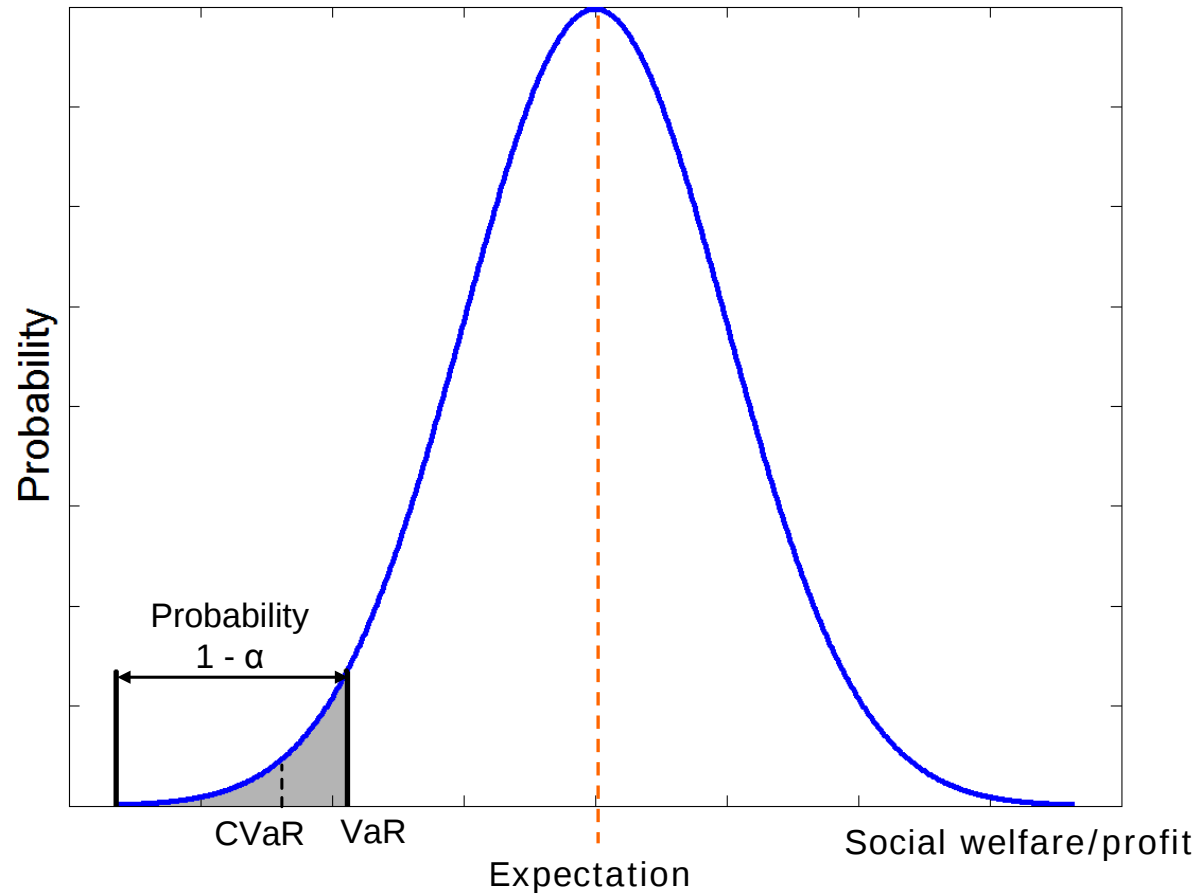
- Expected value
- Any other quantile

being

Function that takes a random variable and returns a real value

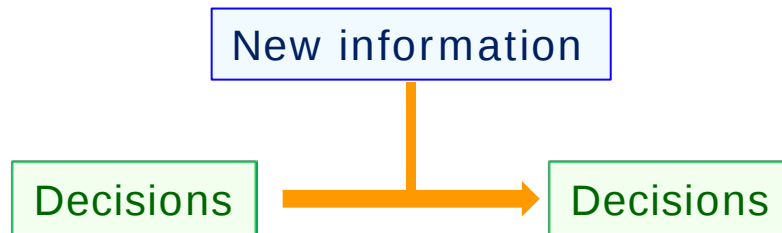
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Stochastic Programming with Recourse



Stochastic Programming with Recourse

Two-stage setting:



Stochastic Programming with Recourse

- *Multi-stage stochastic problems* deal with models in which this “decide-observe-decide” pattern is repeated more than one.

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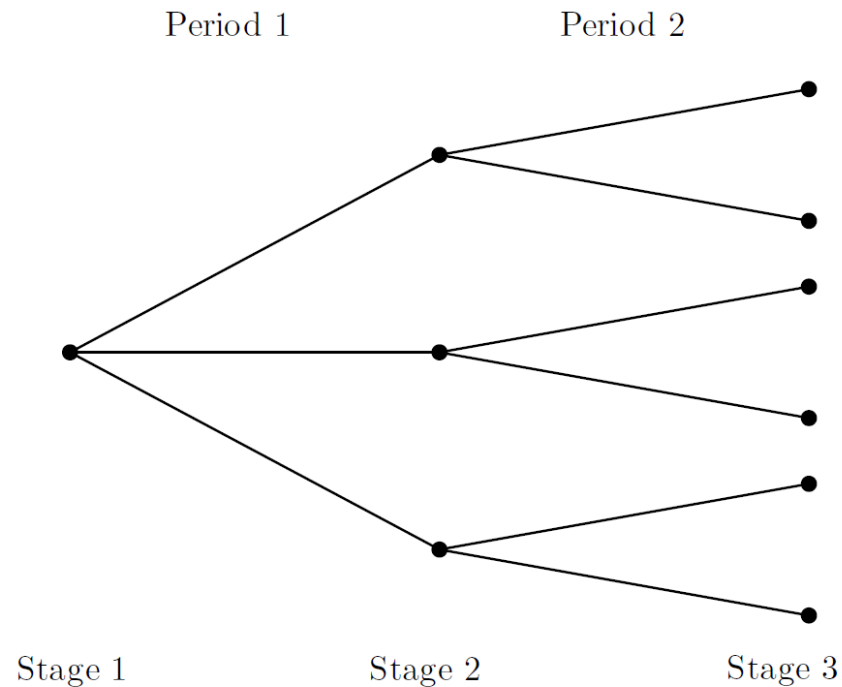
Stochastic Programming with Recourse

Multi-stage setting:



Stochastic Programming with Recourse

Multi-stage tree:



Stochastic Programming with Recourse

- *Multi-stage stochastic problems* deal with models in which this “decide-observe-decide” pattern is repeated more than one.
 - Modify the scenario generation procedure
 - Non-anticipativity constraints to model the observe-decide-observe pattern.

Chance-constrained Programming

$$\begin{array}{ll}\text{Maximize} & c^T x \\ \text{s. t.} & \boxed{P[Ax \leq b] \geq p} \\ & x \geq 0\end{array}$$

Robust optimization

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + G(x, \omega) \\ \text{s.t.} &&& Ax \leq b \\ &&& x \geq 0 \end{aligned}$$

being

$$G(x, \omega) = \left\{ \begin{array}{ll} \underset{\omega \in \Omega}{\text{Min}} & \max_{y_\omega} q^T y_\omega \\ \text{s.t.} & Wy_\omega \leq h_\omega - T_\omega x \\ & y_\omega \geq 0 \end{array} \right\}, \forall \omega \in \Omega$$

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Worst-case scenario!

The solution remains feasible, thus *robust*, for any realization of the uncertain parameters

Robust optimization

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + G(x, \omega) \\ & \text{s.t.} && Ax \leq b \\ & && x \geq 0 \end{aligned}$$

being

Just the support (range) of uncertain parameters is needed

$$G(x, \omega) = \left\{ \underset{\omega \in \Omega}{\text{Min}} \max_{y_\omega} q^T y_\omega \right. \\ \left. \text{s.t.} \quad \begin{aligned} Wy_\omega &\leq h_\omega - T_\omega x \\ y_\omega &\geq 0 \end{aligned} \right\}, \forall \omega \in \Omega$$

A rough comparison ...

	Stochastic programming with recourse	Chance-constrained programming	Robust optimization
Recourse structure?	Yes	No	Not necessarily
Uncertainty model?	Probability distribution	Probability distribution	Deterministic set
Scenarios?	Usually	Not necessarily	No
Feasible?	For every plausible state of nature	"As much as possible"	Immune to uncertainty
Optimal?	Quantile target	Tradeoff: optimality vs. feasibility	Worst-case scenario

To gain insight into ...

Stochastic programming

- J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*. New York: Springer-Verlag, 1997

Robust optimization

- D. Bertsimas and M. Sym, "The price of robustness," *Operations Research*, vol. 52, no. 1, pp. 35–53, 2004

Decision making in electricity markets

- A. J. Conejo, M. Carrión, J. M. Morales, *Decision Making Under Uncertainty in Electricity Markets*, International Series in Operations Research & Management Science, Springer, New York. 2010



Thanks for your attention!