Tightening big-M for Optimal Transmission Switching

FLEXIBASE Workshop (May 5th 2023)

Salvador Pineda

(joint work with J. M. Morales, C. Dominguez, A. Porras)

OASYS group, University of Málaga (Spain)

About Málaga

- Over 300 sundays per year (known as Costa del Sol)
- University of Málaga was stablished in 1972 and currently has 40000 students and 2500 faculty members
- Málaga is becoming the Silicon Valley of the south of Spain
- Andalusia Technology Park includes over 600 companies (Oracle, Ericsson, IBM, TDK, Huawei, Microsoft, Cisco), 20.000 employees and a turnover of 2.000 M€ in 2018

Google To Open A Cybersecurity ‘Centre Of Excellence’ In Malaga

By **Chris King** - 11 February 2021 @ 21:05

About OASYS

Optimization and **A**nalytics for **S**ustainable energ**Y** **S**ystems (2018)

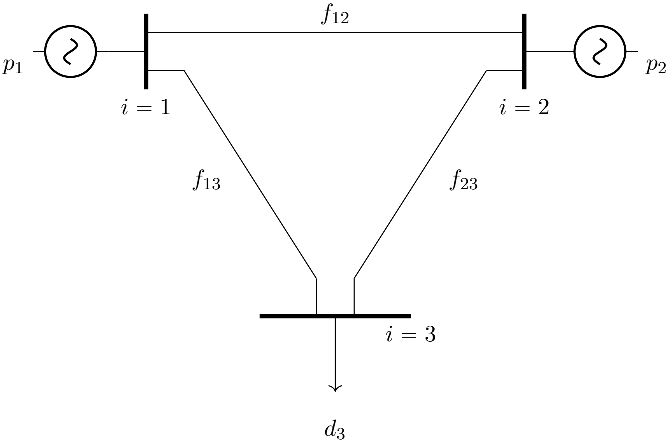
- 3 professors
- 4 Postdoc
- 2 PhD students
- 1 support assistant

Research topics:

- Mathematical models for decision-making under uncertainty
- Use of large amounts of data for Smart Energy Grids
- Forecasting and optimization for Sustainable Energy Systems
- Algorithms to solve large-scale optimization problems

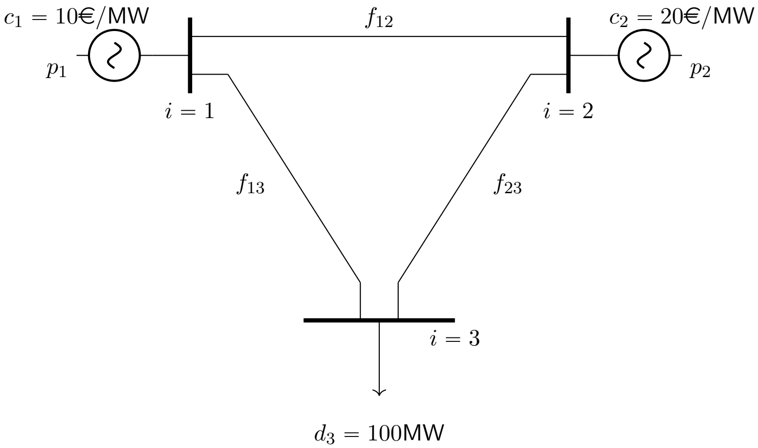
More info: oasys.uma.es

Illustrative example



Illustrative example

If we ignore capacities of units and lines and electricity physics, what are the values of $p_1, p_2, f_{12}, f_{13}, f_{23}$ that satisfy demand at the minimum cost?



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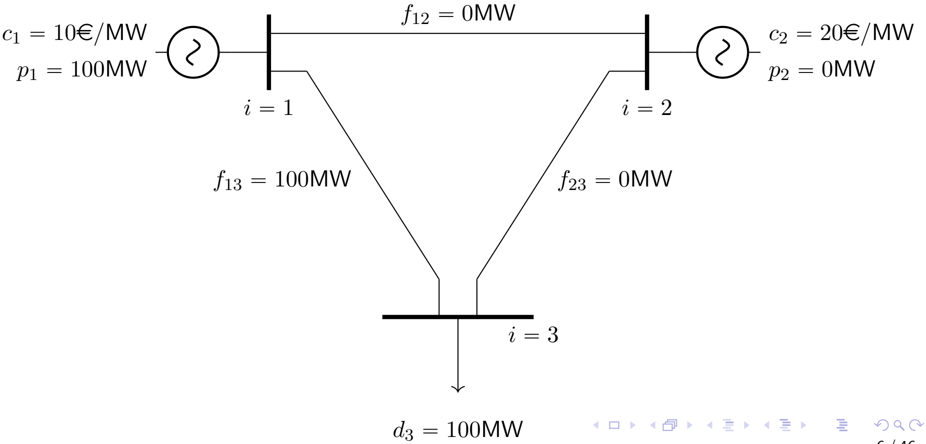
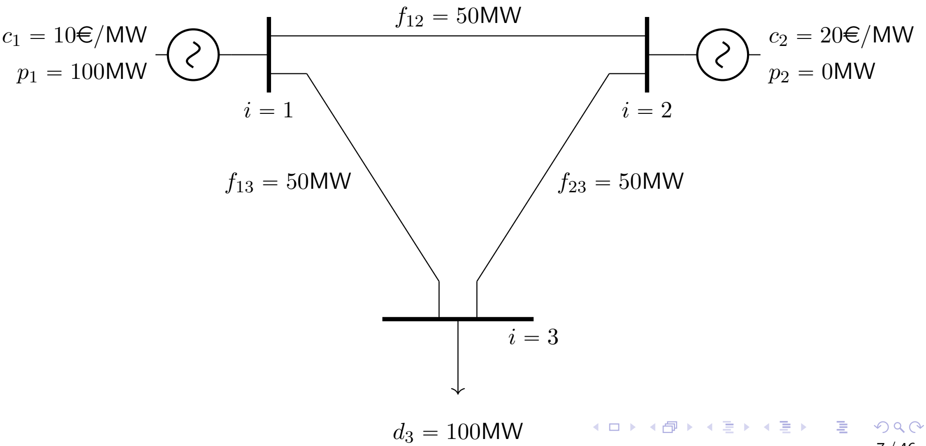
This is one possible with cost of 1000€



Illustrative example

If we ignore capacities of units and lines and electricity physics, what are the values of $p_1, p_2, f_{12}, f_{13}, f_{23}$ that satisfy demand at the minimum cost?

This is another possible with the same cost of 1000€

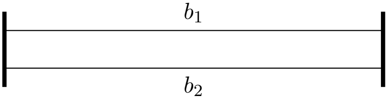


Economic dispatch

$$
\begin{aligned}
\min_{p_i, f_{ij}} \quad & \sum_i c_i \, p_i && \text{(1a)} \\
\text{s.t.} \quad & \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(1b)}
\end{aligned}
$$

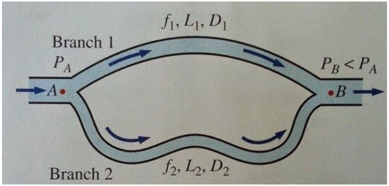
Electrons are not potatoes

If two lines in parallel have the same characteristics, their flows are equal
$b_1 = b_2 \implies f_1 = f_2$



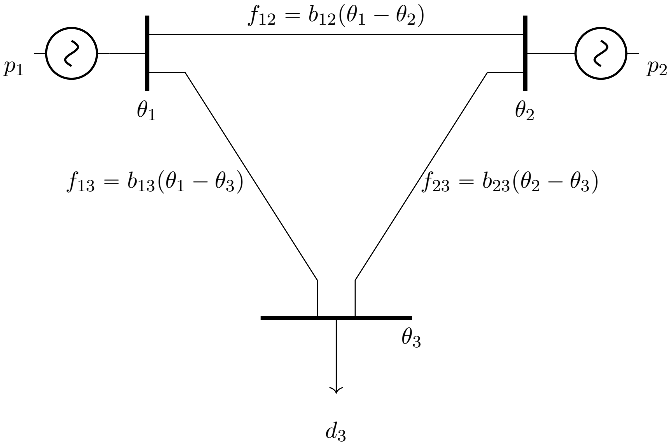
If two lines in parallel are different, their flows are also linked
$b_1 = 2b_2 \implies f_1 = 2f_2$

You can think of it as two parallel water pipes



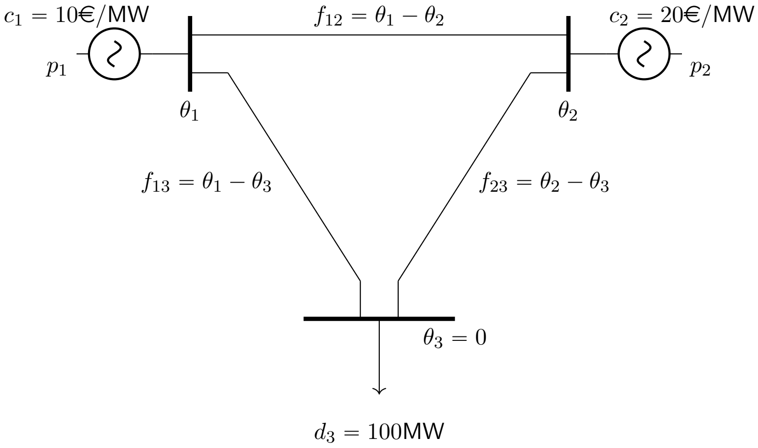
Electrons are not potatoes

Long story short, this condition is imposed by using variables θ_i



Illustrative example

If we ignore capacities of units and lines but consider electricity physics ($b_{12} = b_{13} = b_{23} = 1$), what are the values of $p_1, p_2, f_{12}, f_{13}, f_{23}$?



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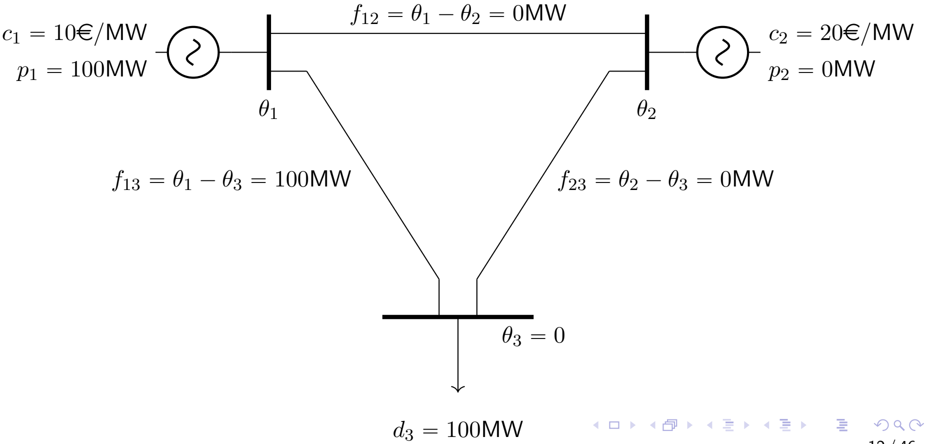
If we ignore capacities of units and lines but consider electricity physics ($b_{12} = b_{13} = b_{23} = 1$), what are the values of $p_1, p_2, f_{12}, f_{13}, f_{23}$?

Is this solution valid?



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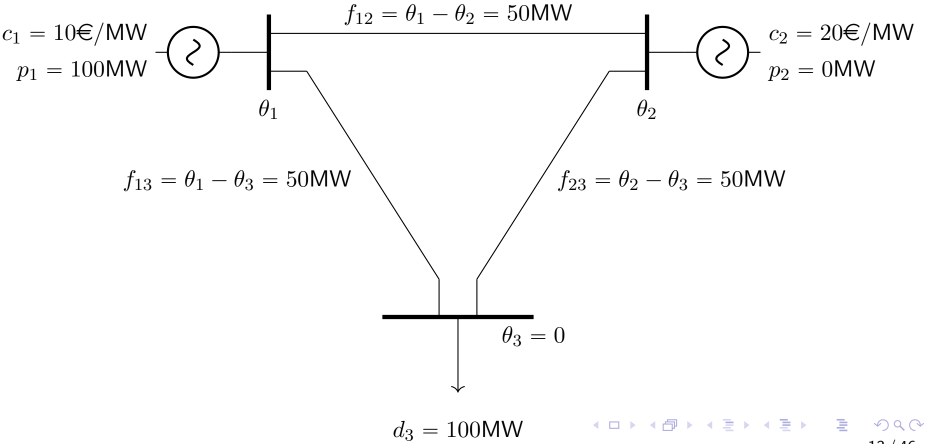
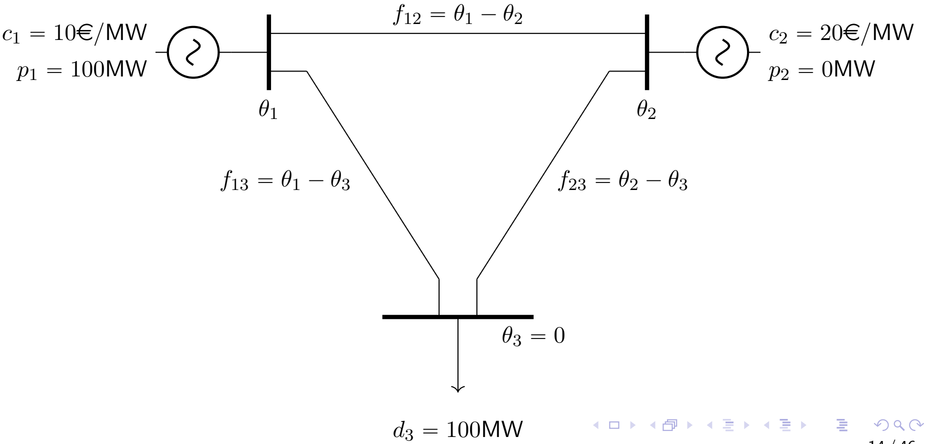


Illustrative example

If we ignore capacities of units and lines but consider electricity physics ($b_{12} = b_{13} = b_{23} = 1$), what are the values of $p_1, p_2, f_{12}, f_{13}, f_{23}$?

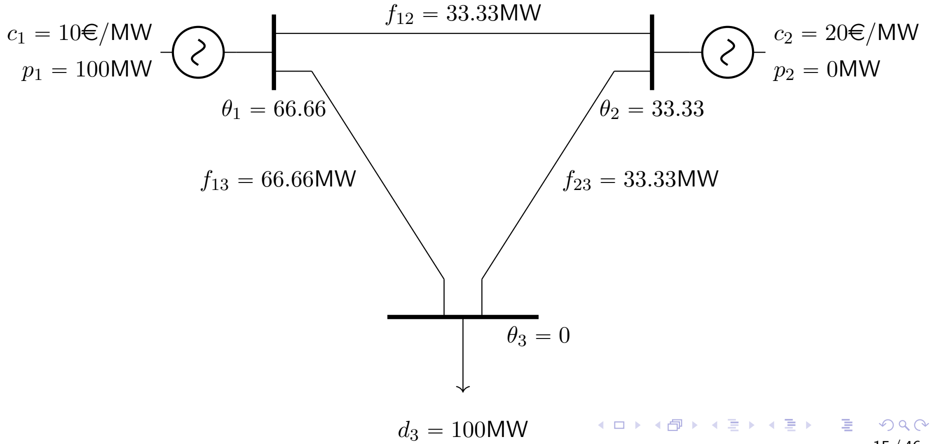
Can you find the optimal solution?



Illustrative example

If we ignore capacities of units and lines but consider electricity physics ($b_{12} = b_{13} = b_{23} = 1$), what are the values of $p_1, p_2, f_{12}, f_{13}, f_{23}$?

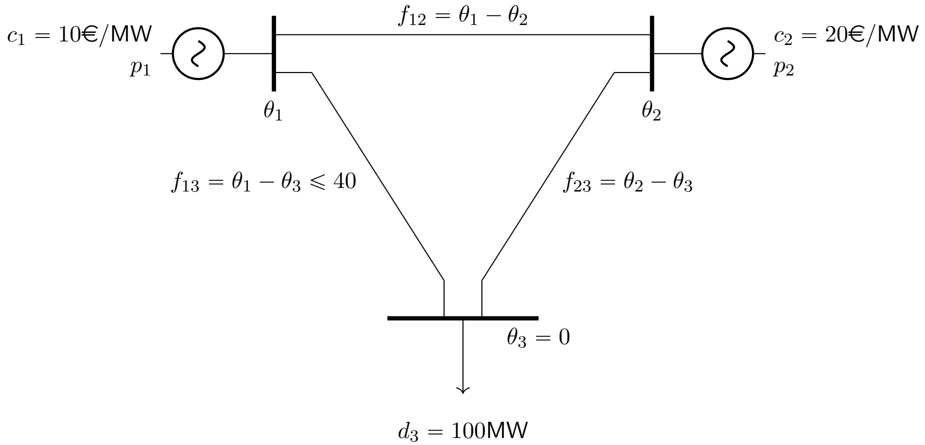
This is the only optimal solution that complies with electricity physics



Illustrative example

If the flow f_{13} cannot be higher than 40MW, what are the values of $p_1, p_2, f_{12}, f_{13}, f_{23}$?

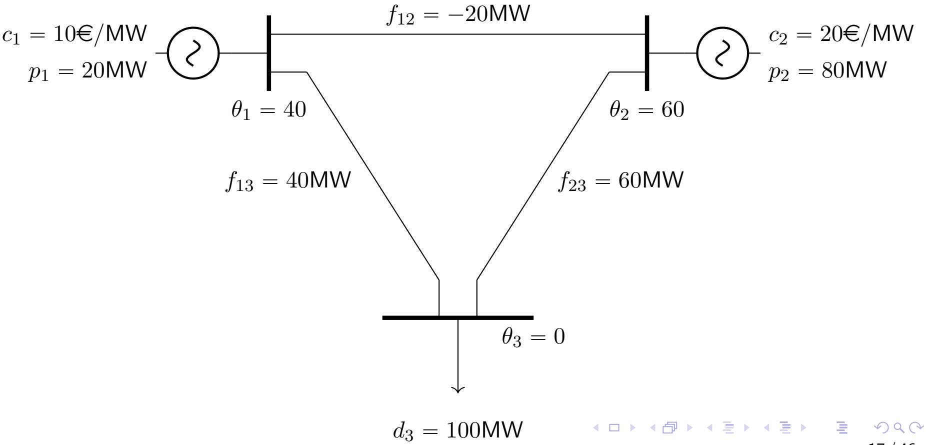
What is the optimal solution now?



Illustrative example

If the flow f_{13} cannot be higher than 40MW, what are the values of $p_1, p_2, f_{12}, f_{13}, f_{23}$?

This is the only optimal solution that complies with electricity physics and the capacity with a cost of 1800€ (80% increase)



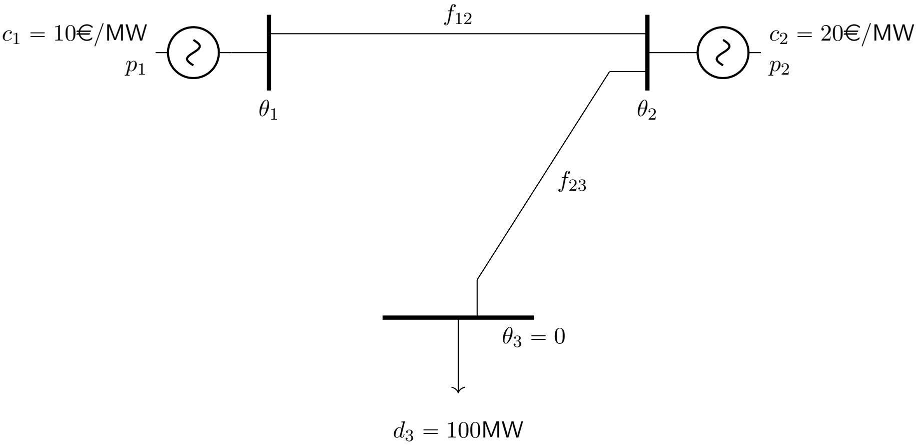
Optimal power flow

$$
\begin{aligned}
\min_{p_i, f_{ij}, \theta_i} \quad & \sum_i c_i \, p_i && \text{(2a)}\\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(2b)}\\
& f_{ij} = b_{ij}(\theta_i - \theta_j), \quad \forall (i,j) \in \mathcal{L} && \text{(2c)}\\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(2d)}\\
& -\underline{f}_{ij} \leqslant f_{ij} \leqslant \overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(2e)}
\end{aligned}
$$

IMPORTANT: If electrons could be "moved" as potatoes, constraint (2c) would not be needed and the problem would be more relaxed!!

Illustrative example

If line 13 is disconnected, what are the values of p_1, p_2, f_{12}, f_{23}?

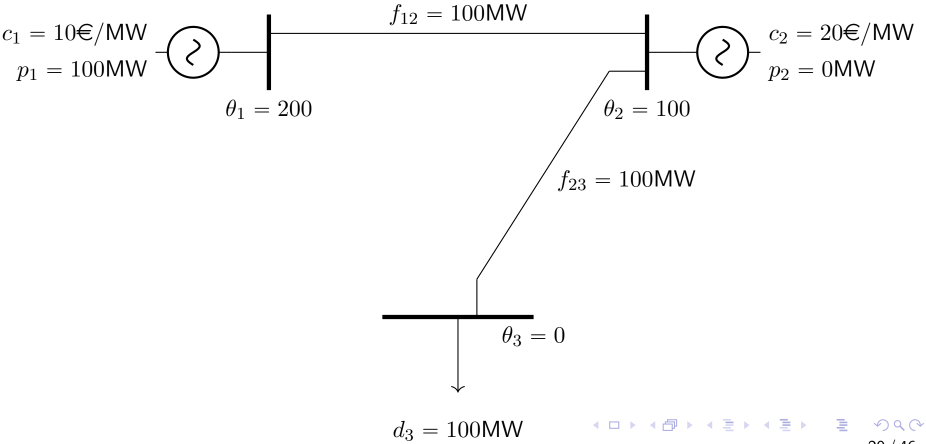


Illustrative example

If line 13 is disconnected, what are the values of p_1, p_2, f_{12}, f_{23}?

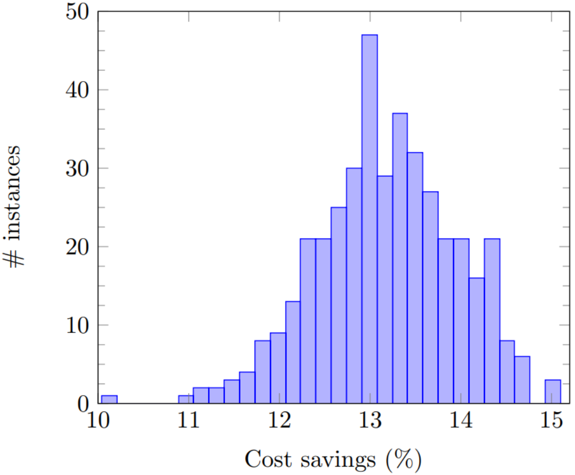
This is the optimal solution now with a cost of 1000€.

Disconnecting lines can reduce cost!!!



Optimal transmission switching

In the 118-bus system, the average cost saving is 13.2%



Optimal transmission switching

$$\min_{p_i, f_{ij}, \theta_i, x_{ij}} \quad \sum_i c_i \, p_i \tag{3a}$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i \tag{3b}$$

$$f_{ij} = x_{ij} b_{ij} (\theta_i - \theta_j), \quad \forall (i,j) \in \mathcal{L} \tag{3c}$$

$$\underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i \tag{3d}$$

$$-x_{ij} \underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij} \overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} \tag{3e}$$

$$x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in \mathcal{L} \tag{3f}$$

IMPORTANT: We have non-linear terms in (3c)

Optimal transmission switching

To avoid the non-linear terms in

$$f_{ij} = x_{ij} b_{ij}(\theta_i - \theta_j)$$

We replace it by

$$b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij})$$

Together with equation

$$-x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}$$

We have that:

- If $x_{ij} = 1 \Rightarrow b_{ij}(\theta_i - \theta_j) \leqslant f_{ij} \leqslant b_{ij}(\theta_i - \theta_j)$ and $-\underline{f}_{ij} \leqslant f_{ij} \leqslant \overline{f}_{ij}$

- If $x_{ij} = 0 \Rightarrow f_{ij} = 0$ and $\underline{M}_{ij} \leqslant b_{ij}(\theta_i - \theta_j) \leqslant \overline{M}_{ij}$

Optimal transmission switching

$$
\begin{aligned}
\min_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & \sum_i c_i \, p_i && \text{(4a)} \\
\text{s.t.} \quad & \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(4b)} \\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(4c)} \\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(4d)} \\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(4e)} \\
& -x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(4f)} \\
& x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in \mathcal{L} && \text{(4g)}
\end{aligned}
$$

$\underline{M}_{ij}$ and $\overline{M}_{ij}$ must be valid bounds of $b_{ij}(\theta_i - \theta_j)$ when $x_{ij} = 0$

If $\underline{M}_{ij}$ and $\overline{M}_{ij}$ are too large, the relaxation of the MIP problem lead to very bad bounds and the computational burden increases

Optimal transmission switching

Since $\underline{M}_{ij} \leqslant b_{ij}(\theta_i - \theta_j) \leqslant \overline{M}_{ij}$ when $x_{ij} = 0$, we can compute these bounds for a particular switchable line (i', j') as

$$
\begin{aligned}
\overline{M}^{\mathrm{OPT}}_{i'j'} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & b_{i'j'}(\theta_{i'} - \theta_{j'}) && \text{(5a)}\\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(5b)}\\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(5c)}\\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(5d)}\\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(5e)}\\
& -x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(5f)}\\
& x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in \mathcal{L} && \text{(5g)}\\
& x_{i'j'} = 0 && \text{(5h)}
\end{aligned}
$$

However, this problem is as difficult as the original one!!

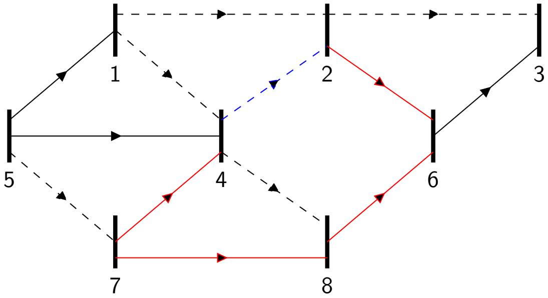
Optimal transmission switching

Fattahi et. al (2019) find a bound on $\overline{M}_{ij}^{\text{OPT}}$ if there exists a connected spanning subgraph of the network with non-switchable lines

$$\overline{M}_{i'j'}^{\text{OPT}} \geqslant b_{i'j'} \sum_{(k,l)\in SP_{i'j'}} \frac{\overline{f}_{kl}}{b_{kl}}$$

where $SP_{i'j'}$ is the shortest path between nodes i' and j' (very easy to compute using Dijkstra's algorithm)

Optimal Transmission Switching



$$\overline{M}_{42} \geqslant b_{42}(\theta_4 - \theta_2) \text{ if } x_{42} = 0$$

$$(\theta_4 - \theta_2) = (\theta_4 - \theta_7) + (\theta_7 - \theta_8) + (\theta_8 - \theta_6) + (\theta_6 - \theta_2)$$

$$b_{47}(\theta_4 - \theta_7) \leqslant \overline{f}_{47} \implies (\theta_4 - \theta_7) \leqslant \overline{f}_{47}/b_{47}$$

$$(\theta_4 - \theta_2) \leqslant \overline{f}_{47}/b_{47} + \overline{f}_{78}/b_{78} + \overline{f}_{86}/b_{86} + \overline{f}_{62}/b_{62}$$

$$\overline{M}_{42} \geqslant b_{42}(\overline{f}_{47}/b_{47} + \overline{f}_{78}/b_{78} + \overline{f}_{86}/b_{86} + \overline{f}_{62}/b_{62})$$

Optimal transmission switching

Instead of solving the original bounding problem to compute $\underline{M}_{ij}^{\text{opt}}$, we solve the following linear relaxation

$$
\begin{aligned}
\overline{M}_{i'j'}^{\text{LR}} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & b_{i'j'}(\theta_{i'} - \theta_{j'}) && \text{(6a)}\\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(6b)}\\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(6c)}\\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(6d)}\\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(6e)}\\
& -x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(6f)}\\
& 0 \leqslant x_{ij} \leqslant 1, \quad \forall (i,j) \in \mathcal{L} && \text{(6g)}\\
& x_{i'j'} = 0 && \text{(6h)}
\end{aligned}
$$

This problem can be "too relaxed" and provide poor bounds, so we include valid inequalities

Optimal transmission switching

Inspired by Porras et. at (2022), we include a bound on the cost using a "reasonable good" feasible solution.

$$\sum_i c_i p_i \leqslant \text{cost}$$

A naive approach is to satisfy the demand with the most expensive units. We denote this cost as cost^{NAI}

One possibility is to compute the cost with all lines connected by solving an OPF problem (LP). We denote this as cost^{OPF}

Another option is using greedy heuristics. For instance, we can start with all lines connected and disconnect the line that involves highest savings by solving several OPF problem. We repeat the process until any disconnection increases the cost. We denote this as cost^{GRE}

Optimal transmission switching

If the bound on the cost is considered, we solve the following problem

$$
\begin{aligned}
\overline{M}^{\mathrm{LR}}_{i'j'} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & b_{i'j'}(\theta_{i'} - \theta_{j'}) && \text{(7a)} \\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(7b)} \\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(7c)} \\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(7d)} \\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(7e)} \\
& -x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(7f)} \\
& 0 \leqslant x_{ij} \leqslant 1, \quad \forall (i,j) \in \mathcal{L} && \text{(7g)} \\
& x_{i'j'} = 0 && \text{(7h)} \\
& \sum_i c_i p_i \leqslant \mathsf{cost}^{\mathrm{NAI/OPF/GRE}} && \text{(7i)}
\end{aligned}
$$

Optimal transmission switching

We also solve bounding problems to improve $\underline{f}_{ij}$ and $\overline{f}_{ij}$ as follows

$$
\begin{aligned}
\overline{f}^{\mathrm{LR}}_{i'j'} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & b_{i'j'}(\theta_{i'} - \theta_{j'}) && \text{(8a)} \\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(8b)} \\
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& 0 \leqslant x_{ij} \leqslant 1, \quad \forall (i,j) \in \mathcal{L} && \text{(8g)} \\
& x_{i'j'} = 1 && \text{(8h)} \\
& \sum_i c_i p_i \leqslant \text{cost}^{\mathrm{NAI/OPF/GRE}} && \text{(8i)}
\end{aligned}
$$

Note that $\overline{f}^{\mathrm{LR}}_{i'j'} \leqslant \overline{f}_{i'j'}$

Optimal transmission switching

We also solve bounding problems to improve $\underline{p}_i$ and $\overline{p}_i$ as follows

$$
\begin{aligned}
\overline{p}_{i'}^{\mathrm{LR}} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & p_{i'} && \text{(9a)}\\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(9b)}\\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(9c)}\\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(9d)}\\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(9e)}\\
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& 0 \leqslant x_{ij} \leqslant 1, \quad \forall (i,j) \in \mathcal{L} && \text{(9g)}\\
& \sum_i c_i p_i \leqslant \mathsf{cost}^{\mathrm{NAI/OPF/GRE}} && \text{(9h)}
\end{aligned}
$$

Note that $\overline{p}_{i'}^{\mathrm{LR}} \leqslant \overline{p}_{i'}$

The proposed method runs as follows:

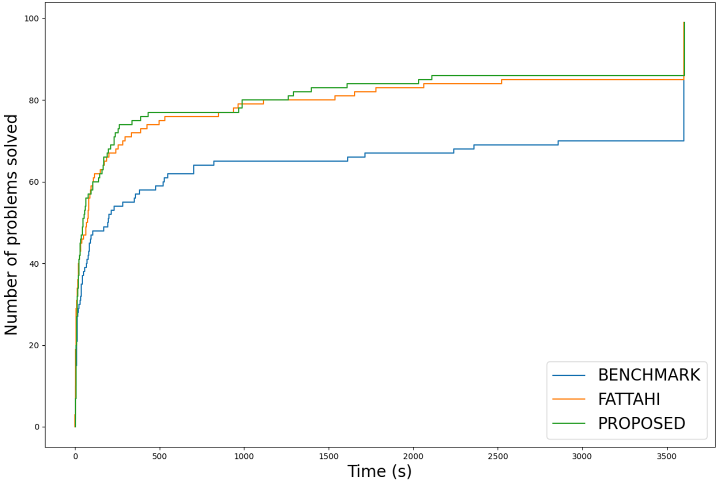
1. Set $\underline{p}_i$, $\overline{p}_i$, $\underline{f}_{ij}$, $\overline{f}_{ij}$ to original capacities
2. Compute $\underline{M}_{ij}^{\text{opt}}$, $\overline{M}_{ij}^{\text{opt}}$ using Fattahi's method
3. Compute the cost bound cost$^{\text{NAI}}$, cost$^{\text{OPF}}$ or cost$^{\text{GRE}}$
4. Solve $2N_G + 2N_L + 2N_S$ bounding problems
5. Repeat step 4 if needed
6. Solve the OTS problem with improved bounds and maximum cost

Numerical simulations

- 118-bus system with 186-lines
- 100 instances with different demands and connected subgraphs
- Each instance includes 69 switchable lines
- Two rounds of solving the bounding problems
- Total time = bounding problems and final OTS problem
- GAP at 0.01% and maximum time 1 hour

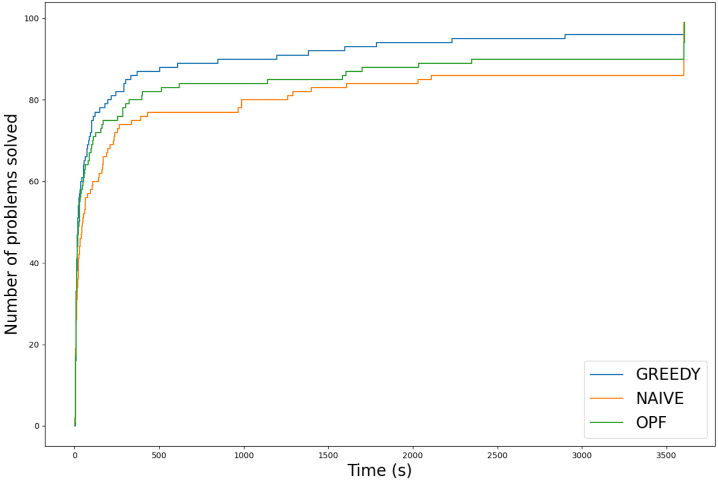
Numerical simulations

Does it makes sense to solve bounding problems?



Numerical simulations

What is the impact of the cost bound?



Numerical simulations

Can we include more valid inequalities?

$$x_1, x_2, x_3 \in \{0, 1\}$$
$$0.5x_1 + 1.5x_2 + 2.5x_3 \geqslant 1$$
$$\min(0.5, 1)x_1 + \min(1.5, 1)x_2 + \min(2.5, 1)x_3 + \geqslant 1$$
$$0.5x_1 + x_2 + x_3 \geqslant 1$$

x_1	x_2	x_3	Original	Rounded	x_1	x_2	x_3	Original	Rounded
0	0	0	x	x	1	1	0	✓	✓
1	0	0	x	x	1	0	1	✓	✓
0	1	0	✓	✓	0	1	1	✓	✓
0	0	1	✓	✓	1	1	1	✓	✓

The integer solutions satisfying both inequalities are the same

Numerical simulations

Can we include more valid inequalities?

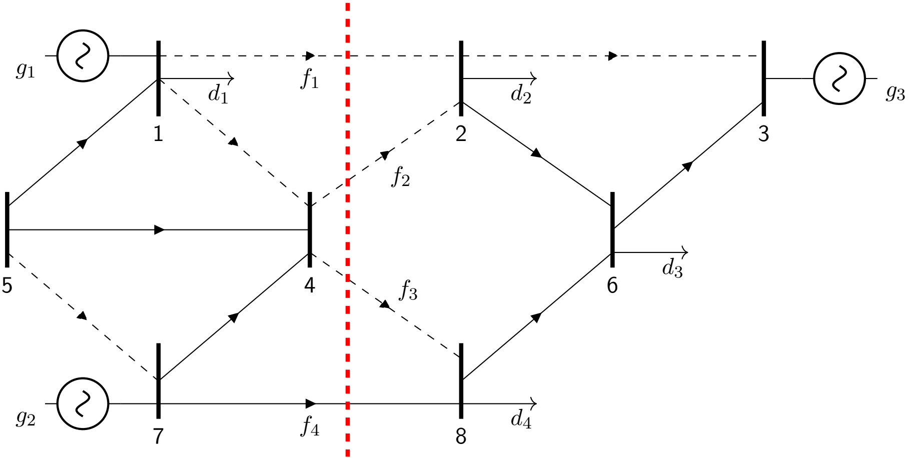
$$x_1, x_2, x_3 \in \{0, 1\}$$
$$0.5x_1 + 1.5x_2 + 2.5x_3 \geqslant 1$$
$$\min(0.5, 1)x_1 + \min(1.5, 1)x_2 + \min(2.5, 1)x_3 + \geqslant 1$$
$$0.5x_1 + x_2 + x_3 \geqslant 1$$

x_1	x_2	x_3	Original	Rounded	x_1	x_2	x_3	Original	Rounded
0	0	0	x	x	1	1	0	✓	✓
1	0	0	x	x	1	0	1	✓	✓
0	1	0	✓	✓	0	1	1	✓	✓
0	0	1	✓	✓	1	1	1	✓	✓
0	0.4	0.4	✓	x	0.4	0.2	0.2	✓	x

The valid inequalities cuts off fractional solutions: TIGHTER formulation.
In integer programming, these are called mixed-integer rounding cuts

Numerical simulations

Can we include more valid inequalities?



$$p_3 + f_1 + f_2 + f_3 + f_4 = d_2 + d_3 + d_4$$

Numerical simulations

Can we include more valid inequalities?

$$
\begin{aligned}
&p_3 + f_1 + f_2 + f_3 + f_4 = d_2 + d_3 + d_4 \\
&f_1 + f_2 + f_3 = d_2 + d_3 + d_4 - p_3 - f_4 \\
&\max(f_1 + f_2 + f_3) \geqslant \min(d_2 + d_3 + d_4 - p_3 - f_4) \\
&\max(f_1 + f_2 + f_3) \geqslant d_2 + d_3 + d_4 - \max(p_3) - \max(f_4) \\
&\overline{f}_1 x_1 + \overline{f}_2 x_2 + \overline{f}_3 x_3 \geqslant d_2 + d_3 + d_4 - \overline{p}_3 - \overline{f}_4 \\
&D = d_2 + d_3 + d_4 - \overline{p}_3 - \overline{f}_4 \\
&\overline{f}_1 x_1 + \overline{f}_2 x_2 + \overline{f}_3 x_3 \geqslant D \\
&\text{If } D > 0 \text{ and } \overline{f}_1, \overline{f}_2, \overline{f}_3 \geqslant 0 \\
&\min(\overline{f}_1, D) x_1 + \min(\overline{f}_2, D) x_2 + \min(\overline{f}_3, D) x_3 \geqslant D
\end{aligned}
$$

We denote these cuts as CUTS1

Numerical simulations

Can we include more valid inequalities?

$$p_3 + f_1 + f_2 + f_3 + f_4 = d_2 + d_3 + d_4$$
$$f_1 + f_2 + f_3 = d_2 + d_3 + d_4 - p_3 - f_4$$
$$\max(f_1 + f_2 + f_3) \geqslant \min(d_2 + d_3 + d_4 - p_3 - f_4)$$
$$\max(f_1 + f_2 + f_3) \geqslant d_2 + d_3 + d_4 - \max(p_3 + f_4)$$
$$\hat{D} = d_2 + d_3 + d_4 - \max(p_3 + f_4)$$
$$\max(p_3 + f_4) \leqslant \max(p_3) + \max(f_4)$$
$$\hat{D} \geqslant D$$
$$\overline{f}_1 x_1 + \overline{f}_2 x_2 + \overline{f}_3 x_3 \geqslant \hat{D}$$

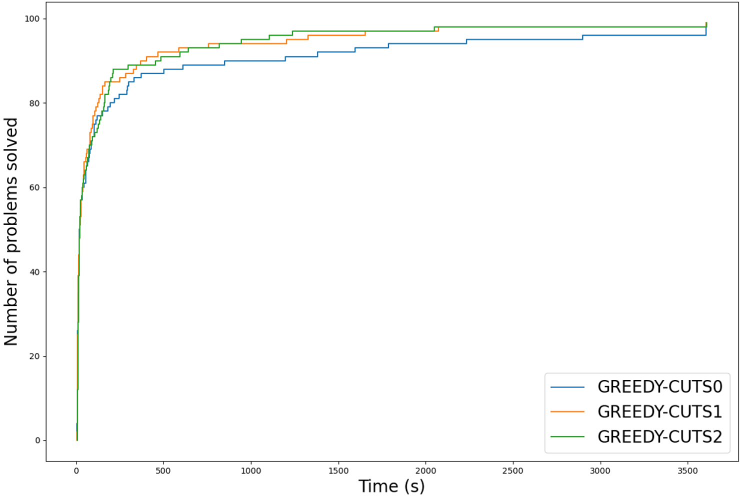
If $\hat{D} > 0$ and $\overline{f}_1, \overline{f}_2, \overline{f}_3 \geqslant 0$

$$\min(\overline{f}_1, \hat{D}) x_1 + \min(\overline{f}_2, \hat{D}) x_2 + \min(\overline{f}_3, \hat{D}) x_3 \geqslant \hat{D}$$

We solve extra bounding problems but we get tighter valid inequalities
We denote these cuts as CUTS2

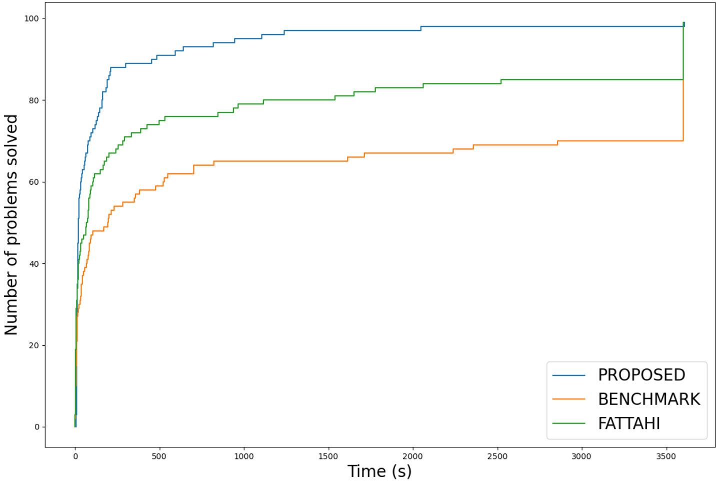
Numerical simulations

Can we include more valid inequalities?



Numerical simulations

How much are we improving things?



Numerical simulations

How much are we improving things?

	Average time (s)	Average gap	Maximum gap
BENCH	1271	0.233%	3.33%
FATTAHI	733	0.082%	1.02%
PROPOSED	203	0.011%	0.19%

We obtain the following improvements with respect to Fattahi:

- The average computational time is reduced a 72%
- The average gap is reduced one order of magnitude
- The maximum gap is also reduced one order of magnitude

Next steps

- Using other methods to find better feasible solutions can also improve the performance of our methodology.
- Add more valid inequalities to further improve the bounds.
- Any other ideas?

Thanks for the attention!

Questions?

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