Scenario Reduction for the Two-Stage Stochastic Unit Commitment Problem

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Motivation

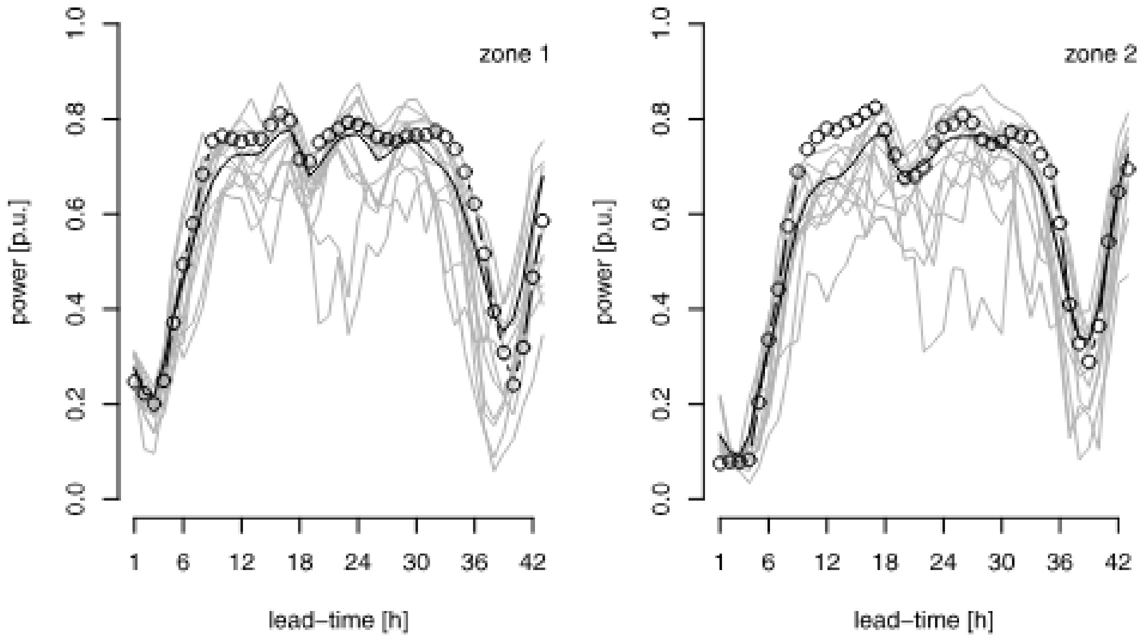


Figure 1: Schematic probabilistic forecast.

- Variety of potentially correlated uncertainty sources
- Represent uncertainty by a stochastic program (SP) $\Rightarrow$ Solving is challenging!
- One solution: Derive a reduced distribution using scenario reduction

Full distribution → Scenario reduction → Reduced distribution

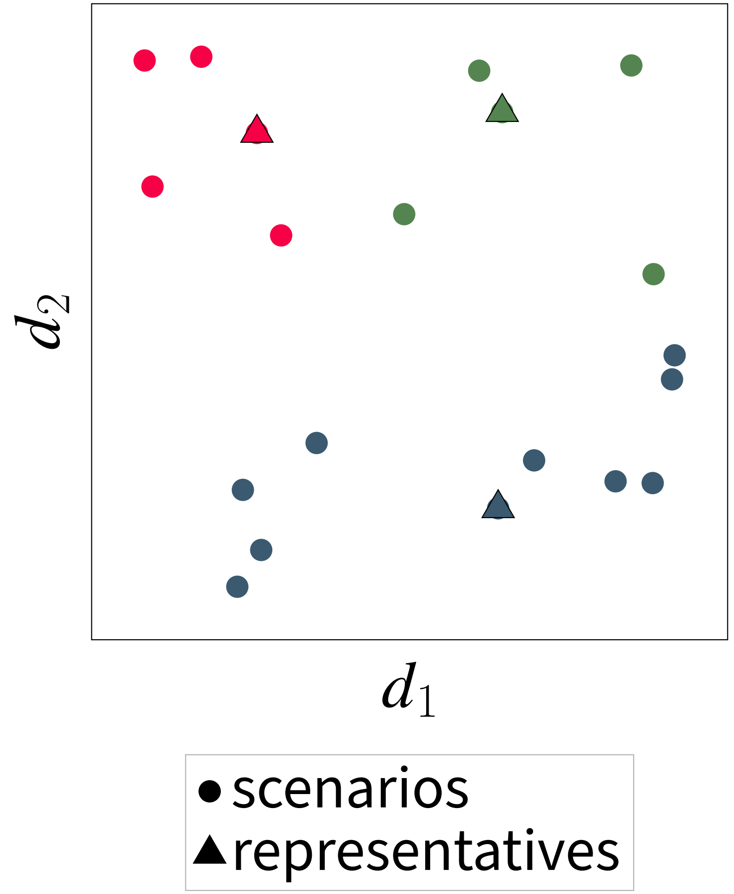
Full distribution → Full SP

Reduced distribution → Reduced SP

Full SP ← compare optimal first stage decisions → Reduced SP

Pinson, P. (2013). Wind Energy: Forecasting Challenges for Its Operational Management. Statistical Science 28(4), 564–585. doi: 10.1214/13-STS445

Scenario reduction for stochastic programming



In a mathematical nutshell

Goal: Based on a discrete sample distribution $\mathbb{P}_n$ supported on a set of scenarios $i \in \mathcal{I} = \{1, \ldots, n\}$ with probabilities p_i and a stochastic problem $\mathbf{SP}$:

- find a distribution $\mathbb{Q}_m$ supported on a set of scenarios $j \in \mathcal{J} = \{1, \ldots, m\}$ with probabilities q_j,
- that has a smaller support, i.e., $m < n$ and ideally $m \ll n$,
- and approximates $\mathbb{P}$ *well* w.r.t. the optimal first-stage solution of $\mathbf{SP}$.

Stochastic programming preliminaries

General two-stage stochastic problem

Consider stochastic programs of form:

$$\min_{x \in \mathcal{X}} z(x, \xi) = f(x) + \sum_{i \in \mathcal{I}} p_i G(x, \xi_i), \tag{1}$$

where the function $Q(x, \xi_i)$ is given by:

$$\min_{v \geq 0} \left\{ a^\top v \;\middle|\; Wv \geq h^{\xi_i} - Tx \right\}. \tag{2}$$

Single scenario problems

Equivalent Problem (1) for single scenario ξ_i:

$$\min_{x \in \mathcal{X}} z(x, \xi_i) = f(x) + G(x, \xi_i), \tag{3}$$

and given first-stage solution $\hat{x} \in \mathcal{X}$:

$$z^*(\hat{x}, \mathbb{P}) = f(\hat{x}) + \sum_{i \in \mathcal{I}} p_i G(\hat{x}, \xi_i). \tag{4}$$

Relative Approximation Error (RAE)

$$\mathrm{RAE}(\mathbb{P}_n, \mathbb{Q}_m) = \frac{z^*(x^*(\mathbb{Q}_m), \mathbb{P}_n) - z^*(\mathbb{P}_n)}{z^*(\mathbb{P}_n)}, \tag{6}$$

Similarity of distributions and scenario reduction

Optimal mass transport problem

$$\mathfrak{D}(\mathbb{P},\mathbb{Q}) = \min_{\pi \in \mathbb{R}_+^{n\times m}} \quad \sum_{i=1}^{n}\sum_{j=1}^{m} \pi_{ij} c(\xi_i, \zeta_j)$$

$$\text{s.t.} \quad \sum_{j=1}^{m} \pi_{ij} = p_i, \quad \forall i \in \mathcal{I},$$

$$\sum_{i=1}^{n} \pi_{ij} = q_j, \quad \forall j \in \mathcal{J},$$

Question: How to choose cost function c?

Algorithm Forward Selection
Dupačová et al., 2003

inputs $\mathbb{P}_n, m, c(\xi_i, \xi_j)$
set $\mathcal{J} \leftarrow \{\}$
Part 1
while $|\mathcal{J}| < m$ **do**
    set $\mathcal{R} \leftarrow \mathcal{I} \setminus \mathcal{J}$
    $j = \arg\min_{j' \in \mathcal{R}} \sum_{i \in \mathcal{R}\setminus\{j'\}} p_i \min_{j'' \in \mathcal{J}\cup\{j'\}} c(\xi_i, \xi_{j''})$
    $\mathcal{J} \leftarrow \mathcal{J} \cup \{j\}$
end while
Part 2
for $j \in \mathcal{J}$ **do**
    $\mathcal{I}_j := \{i \in \mathcal{I} \setminus \mathcal{J} : j = \arg\min_{j' \in \mathcal{J}} c(\xi_i, \xi_{j'})\}$
    update $q_j = p_j + \sum_{i \in \mathcal{I}_j} p_i$
end for
return $\mathbb{Q}_m$

Transportation cost function formulations

Transportation cost functions

- **Input-data-driven**
  - $c^{\mathrm{ID}}(\xi_i, \xi_j) = \|h^{\xi_i} - h^{\xi_j}\|_2^2$
- **Optimization-problem-driven**
  - Morales et al. (2009): $c^{\mathrm{Mo}}(\xi_i, \xi_j) = |z(x^*(\bar{\xi}), \xi_i) - z(x^*(\bar{\xi}), \xi_j)|$, where $\bar{\xi} = \mathbb{E}_{\mathbb{P}_n}[\xi]$
  - Bruninx and Delarue (2016): $c^{\mathrm{Br}}(\xi_i, \xi_j) = |z(x^*(\xi_i), \xi_i) - z(x^*(\xi_j), \xi_j)|$
  - Bertsimas and Mundru (2023): $$2 \cdot c^{\mathrm{Be}}(\xi_i, \xi_j) = z(x^*(\xi_j), \xi_i) - z(x^*(\xi_i), \xi_i) + z(x^*(\xi_i), \xi_j) - z(x^*(\xi_j), \xi_j)$$
  - Our proposal: $c^{\mathrm{Pr}}(\xi_i, \xi_j) = z(x^*(\xi_j), \xi_i) - z(x^*(\xi_i), \xi_i)$
- **Hybrid algorithm:** Use c^{Mo} for pre-grouping and then c^{Pr} for final selection

	ID	Mo	Br	Be	Pr
MIPs	0	1	n	n	n
LPs	0	n	0	$n^2 - n$	$n^2 - n$

Optimal selection with proposed cost function

Theorem

Let $\mathcal{Q}_1 = \{\mathbb{Q}_1^j : j \in \mathcal{I}\}$ denote the family of probability distributions obtained by drawing a single scenario from distribution $\mathbb{P}_n$ and j^* the first scenario drawn using the Forward Selection Algorithm and cost function c^{Pr}. Then the distribution $\mathbb{Q}_1^{j^*}$ minimizes the RAE for $m = 1$, such that $\mathbb{Q}_1^{j^*} = \arg\min_{\mathbb{Q}_1 \in \mathcal{Q}_1} \mathrm{RAE}(\mathbb{P}_n, \mathbb{Q}_1)$.

Loose translation: The first scenario picked by our proposed cost function is the best possible.

Model and case studies

Stochastic unit commitment (UC)

min UC cost + $\sum_{\omega \in \Omega} \pi_\omega$ system operation cost

s.t. UC limits (first stage)

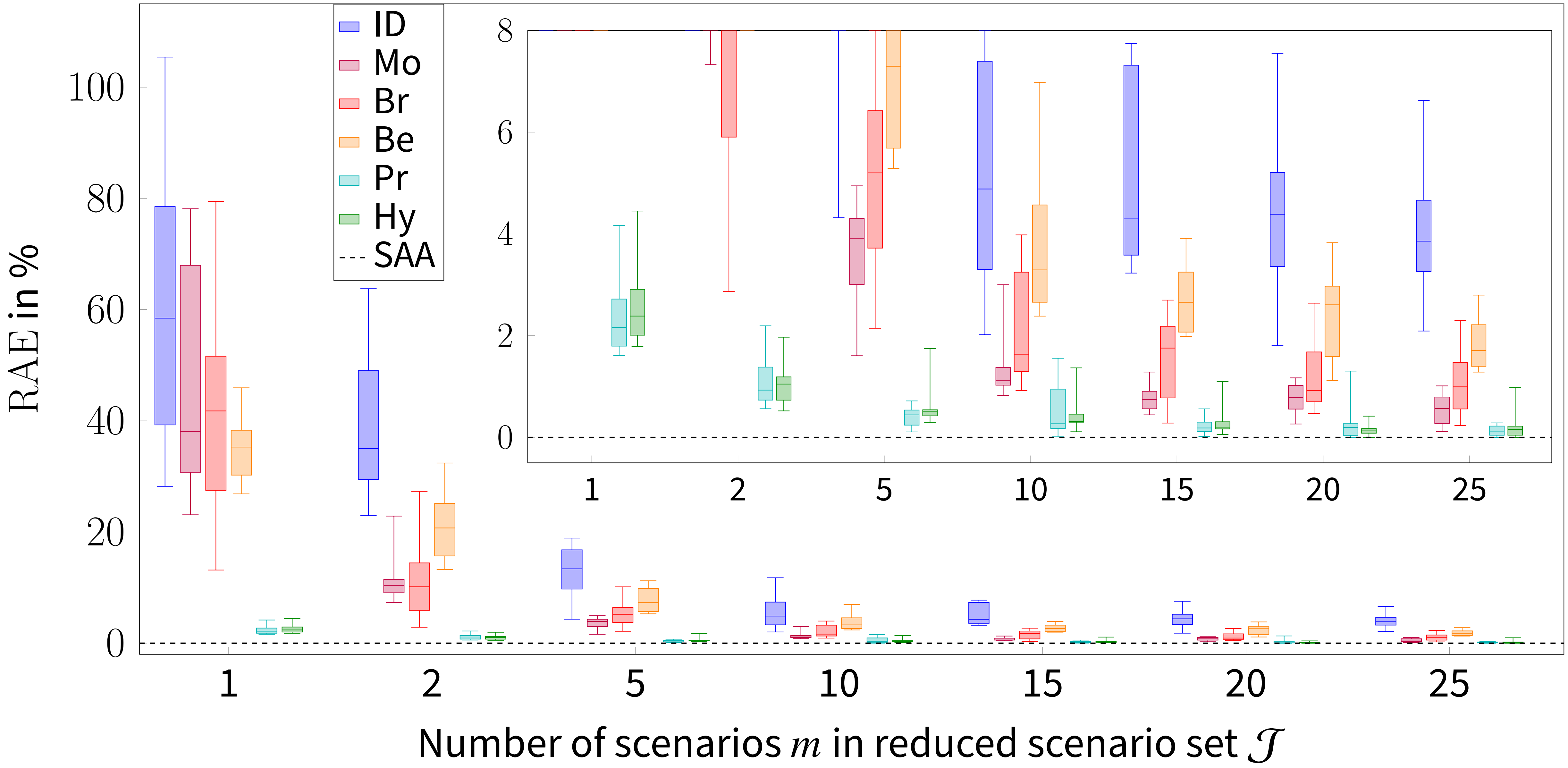
$\forall \omega \in \Omega$: Generator limits (second stage)

$\forall \omega \in \Omega$: DC PF (second stage)

IEEE 300-bus

- 170 committable units, 35 GW total
- 17 wind generators, 26.6 GW total
- MIP gap 1%
- Probabilistic forecast from Pinson (2013)
- $n = 1000$ scenarios, 10 random draws

Results: Solution quality

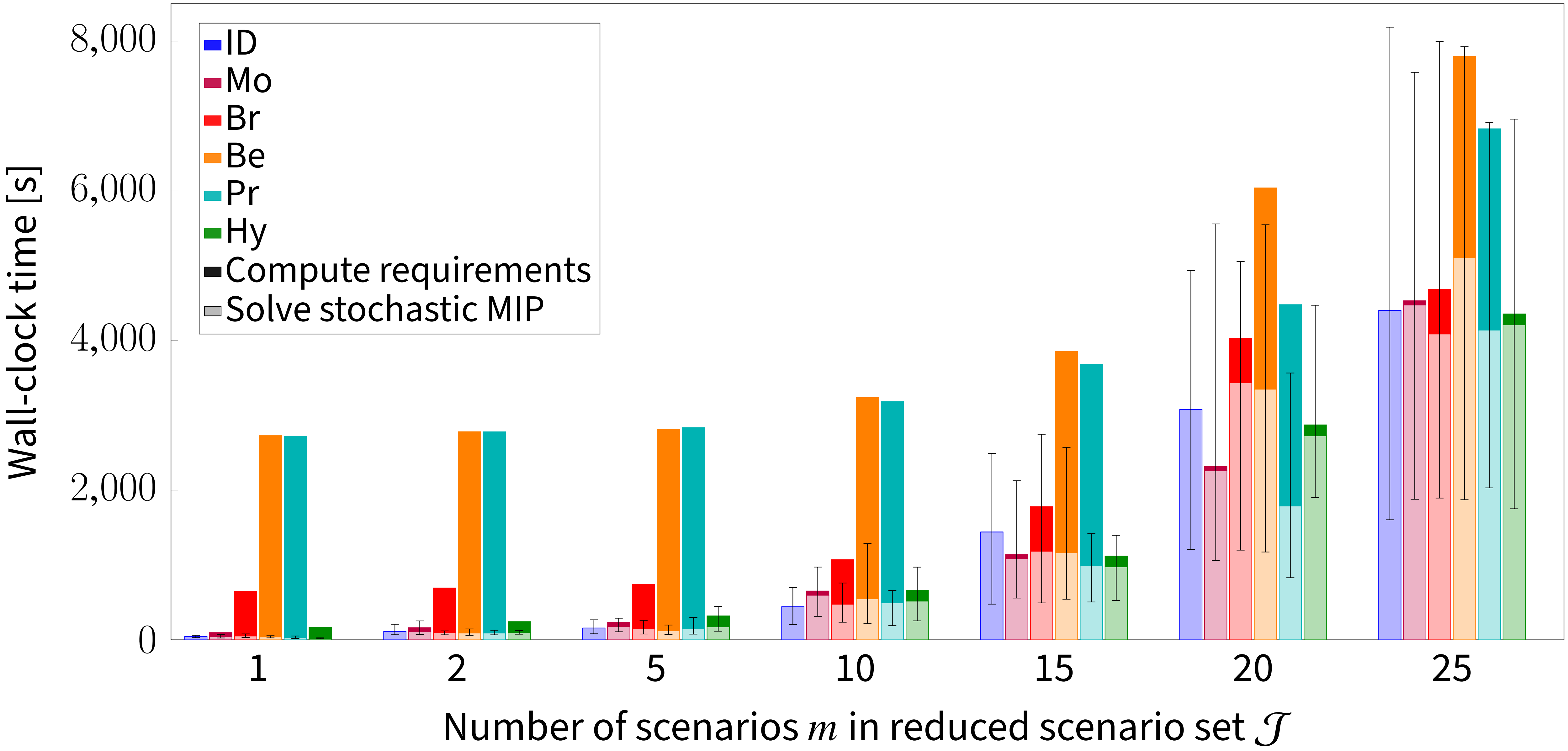


Results: Computational expenses evaluating cost functions

Table 1: Mean Gurobi work units and wall-clock time required to solve problems for evaluating the cost functions for the IEEE 300-bus system, $n = 1000$, over 10 draws.

	Work units			Wall-clock in s		
Cost func. c	MILP	LP	Total	MILP	LP	Total
ID	—	—	—	—	—	—
Mo	56.0	205.0	260.9	33.5	22.6	56.1
Br	46 391.2	—	46 391.2	595.6	—	595.6
Be	46 391.2	153 119.0	199 510.2	595.6	2 093.9	2 689.5
Pr	46 391.2	153 119.0	199 510.2	595.6	2 093.9	2 689.5
Hy	2 396.5	624.1	3 020.6	95.6	50.6	146.2

Results: Combined computational expenses



Conclusions & outlook

- We propose a new cost function for the optimal mass transportation problem.
- Forward Selection is proven to select the optimal scenario first with that function.
- We compare methods on stochastic unit commitment systems with 300 buses.
- Our cost function yields 2.4 % and 0.4 % error with 1 and 5 scenarios.
- Hybrid algorithm cuts solver work 66x and wall-clock time 18x at same accuracy.

Scenario Reduction for Two-Stage Stochastic Mixed-Integer Programs

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Abstract

Two-stage stochastic mixed-integer programs (MIPs) are important tools for decision-making under uncertainty, but representing uncertainty with a large number of scenarios – while ensuring accurate results – can make them challenging to solve. Scenario reduction addresses this by finding a distribution supported on fewer scenarios that still yields similar optimal first-stage decisions. In this paper, we revisit the classical scenario reduction theory ba
distributions and the optimal mass transportation problem. V
transportation cost functions from the literature and propose
lection Algorithm, we prove that our proposed cost function se
a given sample on the first draw with respect to the relative
computational cost of evaluating this cost function, we furth
includes a scenario pre-selection phase. We assess solution qu
on the two-stage stochastic unit commitment problem for smal
ies. With only around five scenarios, the proposed cost functic
optimum to within roughly 2.1 % and 0.4 % error for the sma
hybrid algorithm achieves similar solution quality while reduc
and work (as measured by the Gurobi solver) by a factor of 6(

Keywords: Stochastic programming, Optimal mass transport
Forward selection

PAPER

Thank You!

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Towards time-variant scenario reduction for energy system optimization modeling under uncertainty

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Abstract—Stochastic programming has become a popular tool for supporting decision-making under uncertainty in the long-term planning of energy systems. Existing scenario reduction methods, however, are naive about the long-term temporal nature of scenarios, which limits their efficiency in reducing model size. In this paper, we overcome this inefficiency by proposing a novel time-variant scenario reduction framework that explicitly allows for varying scenario aggregations over time. As a result, scenario probabilities become time-variant, enabling not only the accurate capture of scenario realizations but also their probabilities at the time steps that drive investment decisions. This substantially increases flexibility compared to traditional time-invariant methods, which we demonstrate on a two-stage stochastic generation expansion planning problem with uncertain renewable power production.

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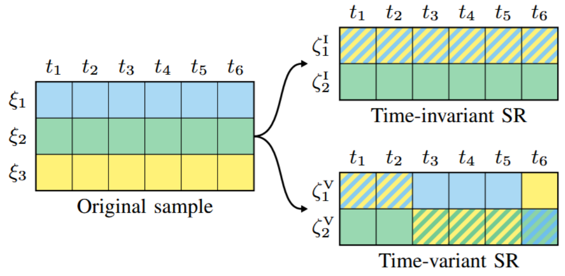

Fig. 1: Schematic representation of the conventional time-invariant (I) and the proposed time-variant (V) scenario reduction (SR) methods with three scenarios and six time steps.

commitment problems in [8], but potentially many scenarios. For large-scale long-term planning ESOMs spanning multiple years, on the other hand, there may be only around 40 historical weather years available, and it may only be possible to include a few of them due to the immense model complexity (e.g., 16 scenarios in [1]). This may render existing time-invariant scenario reduction methods inefficient, as they cannot adjust their marginal distributions over time and are limited by the small number of available scenarios.

At the same time, it facilitates a new branch of research targeting the reduction of a small sample of scenarios with long time horizons by breaking up the temporal chronology of the underlying stochastic process and focusing on capturing the time-index marginal distributions. In this letter, we propose a novel design that allows scenario aggregations to change over time, which we term **time-variant scenario reduction**.