Contextual Robust State Estimation in Distribution Systems

with Scarce Real-Time and Historical Data

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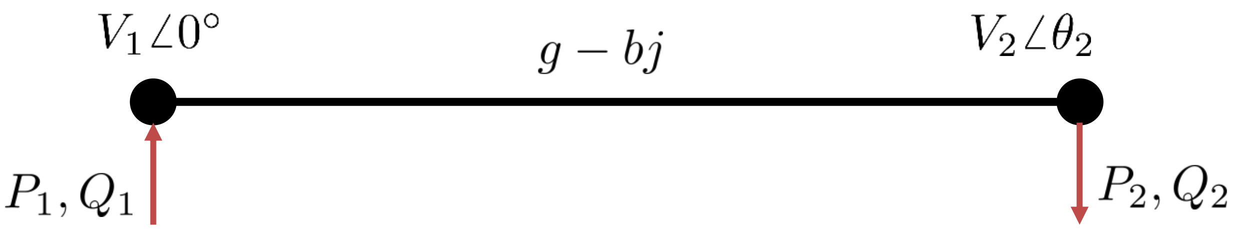
State estimation in a nutshell



$$h_{P_1} = V_1V_2\left(-g\cos(-\theta_2) + b\sin(-\theta_2)\right) + gV_1^2$$
$$h_{Q_1} = V_1V_2\left(-g\sin(-\theta_2) - b\cos(-\theta_2)\right) - bV_1^2$$

$$h_{P_2} = V_2V_1\left(-g\cos\theta_2 + b\sin\theta_2\right) + gV_2^2$$
$$h_{Q_2} = V_2V_1\left(-g\sin\theta_2 - b\cos\theta_2\right) - bV_2^2$$

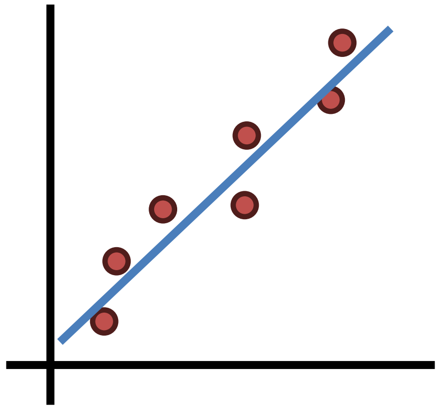
Measurements $\mathbf{z} = [V_1, V_2, \theta_2, P_1, Q_1, P_2, Q_2]^T$

State Vector $\mathbf{x} = [V_1, V_2, \theta_2]^T$

Weighted Least Squares (WLS)

$$\hat{\mathbf{x}} \in \arg\min_{\mathbf{x}} \, (\mathbf{z} - \mathbf{h}(\mathbf{x}))^\top \mathbf{W}(\mathbf{z} - \mathbf{h}(\mathbf{x}))$$

W = diagonal weight matrix (inverse error variances)



With redundancy...

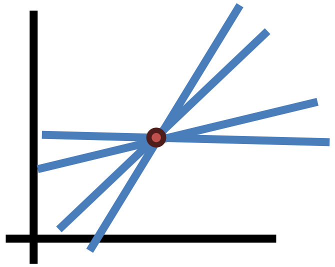
A double information shortage

Shortage #1: Real-Time Data

Fast & Sparse sensors: SCADA, μ-PMUs
Not enough real-time measurements to make the system observable

Slow & Abundant sensors: Smart Meters
Most data arrives with delays

Result:
no observability →



Shortage #2: Historical Data

Historical data is limited, often outdated, and non-representative

Networks evolve: new solar panels, topology changes, EV adoption shifts demand patterns

Result: statistical models trained on past data degrade quickly

$\mathbf{z}^a$ – available in real time

$\mathbf{z}^d$ – delayed, must be inferred from history

Our goal: a state estimator explicitly designed for both shortages simultaneously

Let similar past moments fill the gaps

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1 → **2** → **3**

Observe

Collect available real-time measurements $\mathbf{z}^a$

Search

Find K nearest neighbors in the historical dataset

Estimate

Use neighbors as pseudo-measurements for $\mathbf{z}^d$

Why K-Nearest Neighbors?

- Non-parametric: no model to train, no distributional assumptions
- Contextual: uses current real-time data (the context) to select relevant history
- Data-driven & Interpretable approximation of $P(\mathbf{z}^d|\mathbf{z}^a)$

The baseline: Vanilla K-NN state estimator

WLS-SE on the real-time data and the average of the K pseudo-measurement scenarios

Each delayed measurement in the training set → a plausible "scenario" for the missing data in the test

The weights reflect the sample variance of the neighbors: high variability across neighbors → low confidence → downweighted.

Lazy learning method: no offline training needed

Non-Robust (Vanilla) Estimator

$$\hat{\mathbf{x}}^v_{t'} \in \arg\min_{\mathbf{x}} \, (\mathbf{z}^a_{t'} - \mathbf{h}^a(\mathbf{x}))^\top \mathbf{W}^a (\mathbf{z}^a_{t'} - \mathbf{h}^a(\mathbf{x})) + \frac{1}{K} \sum_{t \in \mathcal{T}^K} (\mathbf{z}^d_t - \mathbf{h}^d(\mathbf{x}))^\top \widehat{\mathbf{W}}^d (\mathbf{z}^d_t - \mathbf{h}^d(\mathbf{x}))$$

$$\text{where } \mathcal{T}^K := \arg\min_{\substack{\mathcal{K} \subseteq \mathcal{T} \\ |\mathcal{K}| = K}} \sum_{k \in \mathcal{K}} (\mathbf{z}^a_{t'} - \mathbf{z}^a_k)^\top \widehat{\mathbf{W}}^a (\mathbf{z}^a_{t'} - \mathbf{z}^a_k)$$

⚠ But wait... This treats pseudo-measurements as if they were reliable. With scarce data, they carry significant statistical uncertainty!

The Contextual Robust State Estimator (CR-SE)

Core idea:
min over states & max over pseudo-measurement weights

$$
\begin{aligned}
\hat{\mathbf{x}}^r_{t'} \in \arg\min_{\mathbf{x}} \max_{w_t \geq 0} & \sum_{t \in \mathcal{T}^T} w_t \left(\mathbf{z}^a_{t'} - \mathbf{h}^a(\mathbf{x})\right)^\top \mathbf{W}^a (\mathbf{z}^a_{t'} - \mathbf{h}^a(\mathbf{x})) \\
& + \sum_{t \in \mathcal{T}^T} w_t \left(\mathbf{z}^d_t - \mathbf{h}^d(\mathbf{x})\right)^\top \widehat{\mathbf{W}}^d (\mathbf{z}^d_t - \mathbf{h}^d(\mathbf{x})) \\
& \text{s.t. } w_t \leq \frac{1}{K}, \ \forall t \in \mathcal{T}^T \quad : \lambda_t \\
& \sum_{t \in \mathcal{T}^T} w_t = 1 \quad : \mu_1 \\
& w_{t+1} - w_t \leq 0, \ \forall t \in \mathcal{T}^T \setminus \{t_{|\mathcal{T}^T|}\} \quad : \phi_t \\
& \sum_{t \in \mathcal{T}^T} w_t \left(\mathbf{z}^a_{t'} - \mathbf{z}^a_t\right)^\top \widehat{\mathbf{W}}^a (\mathbf{z}^a_{t'} - \mathbf{z}^a_t) \leq \rho \quad : \mu_2.
\end{aligned}
$$

- Protects against misspecification of the distribution
- At least *K* neighbors
- Valid distribution with finite discrete support
- Monotonically decresing
- Bounds the contextual distance
- Reformulated into an efficient equivalent single-level minimization problem

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$$\hat{\mathbf{x}}^r_{t'} \in \underset{\mathbf{x},\,\lambda_t,\,\phi_t,\,\mu_1,\,\mu_2}{\arg\min} \quad \frac{1}{K}\sum_{t\in\mathcal{T}^T} \lambda_t + \mu_1 + \rho\,\mu_2$$

$$\begin{aligned}
\text{s.t. } & \lambda_t + \mu_1 + \mu_2\,(\mathbf{z}^a_{t'} - \mathbf{z}^a_t)^\top \mathbf{W}^a (\mathbf{z}^a_{t'} - \mathbf{z}^a_t) + f_t(\phi) \geq \\
& \quad (\mathbf{z}^a_{t'} - \mathbf{h}^a(\mathbf{x}))^\top \mathbf{W}^a (\mathbf{z}^a_{t'} - \mathbf{h}^a(\mathbf{x})) \\
& \quad + (\mathbf{z}^d_t - \mathbf{h}^d(\mathbf{x}))^\top \widehat{\mathbf{W}}^d (\mathbf{z}^d_t - \mathbf{h}^d(\mathbf{x})),\ \forall t \in \mathcal{T}^T \\
& \mu_2 \geq 0 \\
& \lambda_t \geq 0,\ \forall t \in \mathcal{T}^T \\
& \phi_t \geq 0,\ \forall t \in \mathcal{T}^T \setminus \{t_{|\mathcal{T}^T|}\}
\end{aligned}$$

$$f_t(\phi) = \begin{cases} -\phi_t & \text{if } t = t_1 \\ \phi_{t-1} - \phi_t & \text{if } t_1 < t < t_{|\mathcal{T}^T|} \\ \phi_{t-1} & \text{if } t = t_{|\mathcal{T}^T|}. \end{cases}$$

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Overhead: $2|\mathcal{T}| + 1$ variables, $3|\mathcal{T}|$ constraints

Intuition: what does ρ control?

ρ governs the size of the ambiguity set:
¿how much the adversary can deviate from the empirical $1/K$ distribution?

$\rho = \rho_{min}$ $\rho = \rho_{max}$

Small ρ → Trust the data

- Ambiguity set is tight around the $1/K$ distribution.
- Behaves close to the Vanilla estimator.
- Efficient when data is representative.

Large ρ → Hedge your bets

- Wider ambiguity set allows more redistribution.
- Penalizes contextually distant neighbors.
- More conservative: better tail performance.

The sweet spot? We select ρ automatically using online validation

Selecting ρ: Data-driven + online

Define the search range

Grid search between ρ_{min} and ρ_{max}

Validate on the most recent instance

Retrospective state as ground truth

Pick the winner

Min validation error

Solve on the test instance

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Case study setup

IEEE 38-bus

Smaller, tractable baseline

+ IEEE 123-bus

Larger, tests scalability

Time-series with real residential data (Pecan Street)

Variables tested

- SCADA measurements: 1, 3, 5
- Training set size: 10, 20, 50
- Load factor: 0.5, 0.8, 1, 1.2
- 100 test instances per scenario

Metrics

RMSE – Root Mean Squared Error

- Overall estimation accuracy

RCVaR – Root Conditional Value at Risk ($\alpha = 0.95$)

- Tail performance: how bad are the worst 5% of errors?

Benchmarks

Vanilla K-NN + WLS no robustness

Persistent Most recent instance's state

Retrospective All sensors available

Oracle CR-SE validating on test

Results: CR-SE strictly outperforms Vanilla

Baseline scenario: 3 SCADA, Load = 0.5, training = 20, K = 5

	38-bus Network		123-bus Network	
Method	**RMSE ($\times 10^{-3}$)**	**RCVaR ($\times 10^{-3}$)**	**RMSE ($\times 10^{-3}$)**	**RCVaR ($\times 10^{-3}$)**
Vanilla	2.18	6.96	2.21	6.03
CR-SE	**2.13**	**6.70**	**2.16**	**5.62**
Persistent	4.45	14.62	3.69	9.85
Oracle	2.03	6.51	1.91	5.03
Retrospective	1.15	2.63	1.81	4.09

Does CR-SE close the gap between Vanilla and the Oracle?
RMSE = 19–30%, RCVaR = 41–57%

Where CR-SE shines most

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Least real-time measurements

Up to 9% RMSE
Up to 11% RCVaR

For the same context:

- More possible pseudo-measurements
- More different states

Growing training sets

Gap widens with more data

Larger window hurt Vanilla as farther operating conditions are included

CR-SE scales beneficially: it penalizes them via ρ

Heavy loading

Consistent performance across load scenarios

Stronger nonlinearities

Both methods worsen, CR-SE contains the degradation

The performance gap widens precisely in the configurations where the Vanilla is most vulnerable

Key takeaways

1. **Pseudo-measurements have uncertainty – treat it explicitly**
   Existing methods sistematically overlook pseudo-measurements statistical misspecification

2. **Distributionally robust optimization provides a principled fix**
   A min-max formulation hedges against the worst-case reweighting of pseudo-measurements within an ambiguity set

3. **Same tractability as the baseline**
   The robust formulation admits a single-level reformulation with the same structure and comparable computational cost

4. **Best gains where they matter most**
   CR-SE shines under the hardest scenarios: extreme unobservability, larger historical data, and heavy loading

Thank you!

Questions, ideas, collaborations?

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