Solving the Rank Pricing Problem using a Heuristic Approach

Asunción Jiménez-Cordero
asuncionjc@uma.es

Joint work with:
Salvador Pineda Morente
Juan Miguel Morales González

oasys.uma.es

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Outline

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Rank Pricing Problem

Assumptions

- Set of products, $\mathcal{I} = \{1, \ldots, I\}$.
- Set of customers, $\mathcal{K} = \{1, \ldots, K\}$.
- Budgets, b^k, $\forall k \in \mathcal{K}$.
- Preference values, s_i^k, $\forall k \in \mathcal{K}, \forall i \in \mathcal{I}$.
- Customer preferences, $s_{i_1}^k > s_{i_2}^k \Rightarrow$ customer k prefers to buy product i_1 rather than product i_2.

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- **Objective:** determine the **optimal prices** for the products so that the **benefit** of the company and the **satisfaction** of the customers is **maximized** keeping in mind the **budget constraints**.

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- **Mixed-Integer Bilevel Optimization Problem.**

Rank Pricing Problem

Upper level

$$\begin{cases} \max\limits_{\boldsymbol{p}} \sum\limits_{k\in\mathcal{K}} \sum\limits_{i\in\mathcal{I}} p_i x_i^k & \text{(benefit)} \\ \text{s.t. } \; p_i \geq 0, \; i \in \mathcal{I} & \text{(non-negativity)} \end{cases}$$

Lower level, for each customer k

$$\begin{cases} \max\limits_{\boldsymbol{x^k}} \sum\limits_{i\in\mathcal{I}} s_i^k x_i^k & \text{(satisfaction)} \\ \text{s.t. } \; \sum\limits_{i\in\mathcal{I}} x_i^k \leq 1 & \text{(one product at most)} \\ \qquad \sum\limits_{i\in\mathcal{I}} p_i x_i^k \leq b^k & \text{(affordability)} \\ \qquad x_i^k \in \{0,1\}, \; i \in \mathcal{I} & \text{(binary)} \end{cases}$$

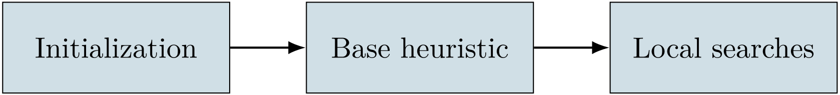
Rank Pricing Problem

- NP-complete, *Rusmevichientong et al. [2006]*.
- Exact approaches, *Bucarey et al. [2021]; Calvete et al. [2019]; Domínguez [2021]*. Valid inequalities and Benders decomposition.
- Heuristic approaches, *Calvete et al. [2024]*. Genetic-based + local search methodology.
- Outstanding results but sometimes difficult, time-consuming and require solving new optimization problems.

- Base heuristic (VNS/genetic) + local search.
- No optimality guarantees but yields good results in a reasonable time.

Outline

General setup



Initialization

- Proposed in *Calvete et al. [2024]*.
- Greedy approach.
- One vector: the richest customer chooses first.
- Very good objective values.
- Number of products $\approx$ Number of customers.
- Alternative: random approach.

Base heuristics (VNS/genetic)

- Based on the concept of neighbor.

k	s_1^k	s_2^k	b^k
1	2	1	18
2	2	1	66
3	1	2	27
4	1	2	34
5	1	2	66
6	2	1	50
7	2	1	42
8	1	2	42

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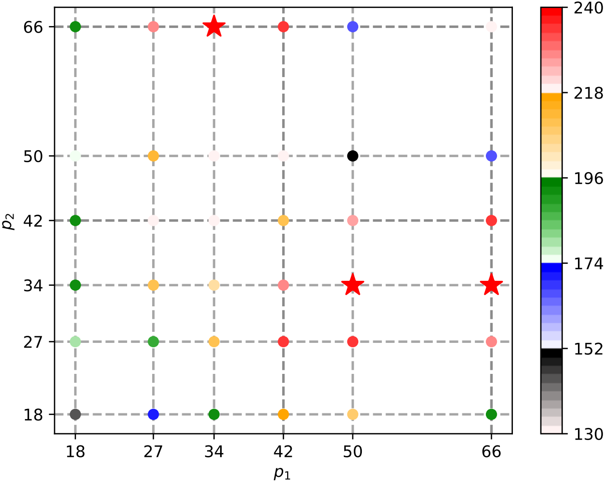
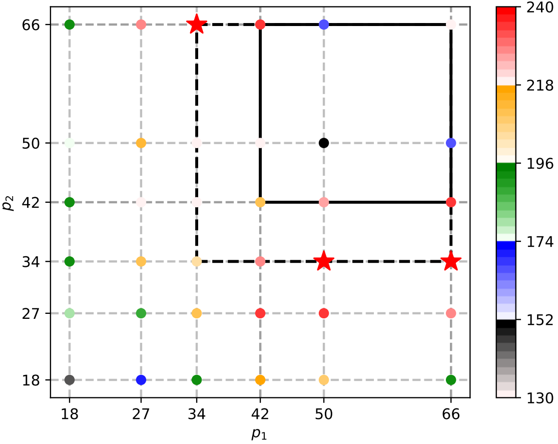
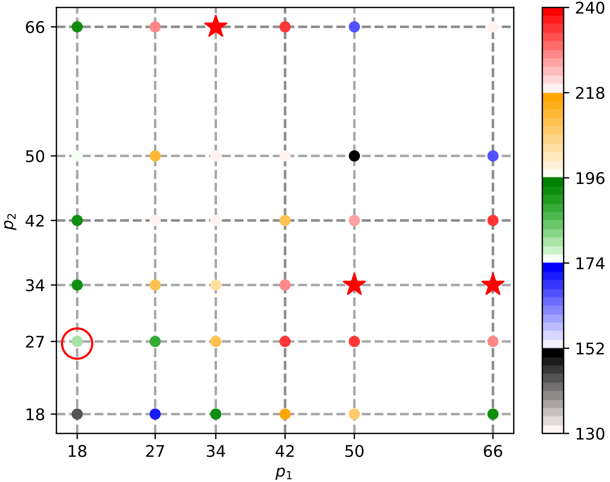


VNS



Genetic



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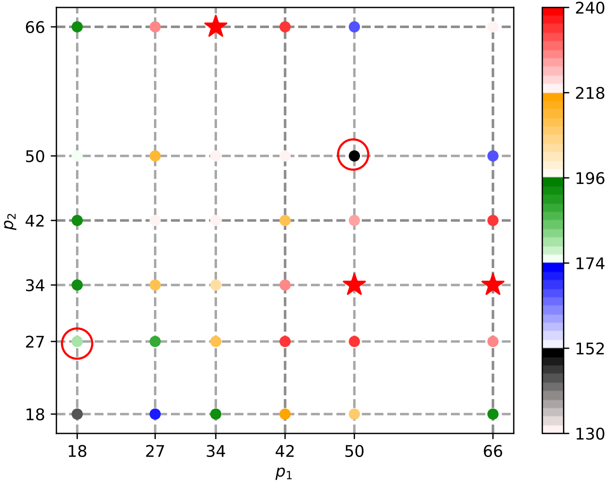

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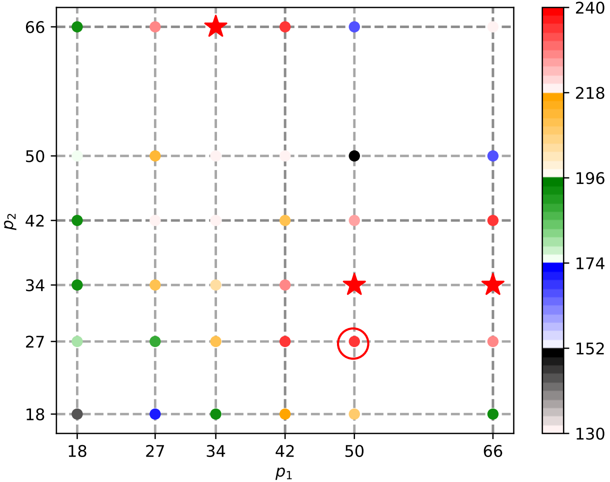
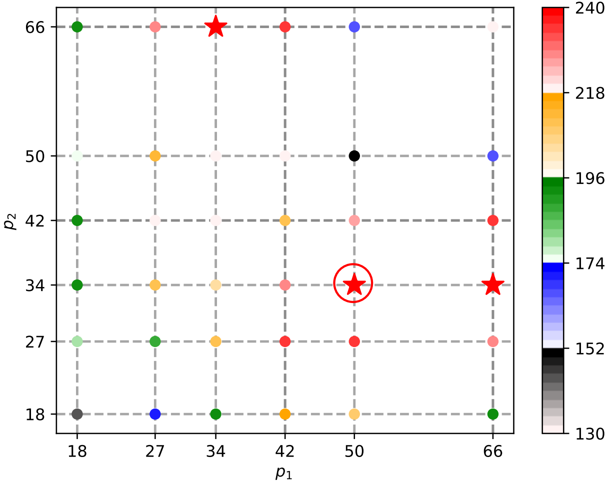


Genetic



Local improvements

Optimization-based local search

- Proposed in *Calvete et al. [2024]*.
- After genetic algorithm.
- Components of vector of products are randomly sorted.
- For each product, we change the budget value, solve the lower-level problem, and evaluate the upper-level objective function.
- We keep the new vector if the objective value improves.

Local improvements

Slack local search

- No need to solve optimization problems.
- Idea: some *slack* between budgets and prices.
- Example: $p = (34, 34)$. Product 1 is chosen by customers 2, 6 and 7 with $b^2 = 66, b^6 = 50$ and $b^7 = 42$.
- Product 1 has some slack. We can update it to $p = (42, 34)$.
- Binary variables do not change.
- Upper-level objective value increases.

Local improvements

Fill local search

- Change in the binary variables.
- For a fixed i, $x_i^k = 0, \forall k \in \hat{\mathcal{K}}$ because price of i is too high.
- *Fill* these gaps.
- Update the price of i by the minimum budget value.
- The upper-level objective value increases.

Local improvements

Two more proposals

- Reassignment local search.
- Conditional reassignment.
- Change in the binary variables.
- Do not need to solve extra optimization problems.

Outline

Setup

- $(K, I) \in \{(30,5),(30,25),(60,50)\}$.
- The data can be downloaded from *OASYS [2024]*.
- Gurobi 10.0.3 (default parameters).
- 1000 runs to get fair conclusions.

Comparative approaches

Exact approach

$$
\begin{cases}
\max\limits_{\boldsymbol{v},\boldsymbol{x}} \sum\limits_{k\in\mathcal{K}}\sum\limits_{i\in\mathcal{I}}\left(\sum\limits_{m\in\mathcal{M}} b^m v_i^m\right) x_i^k \\
\text{s.t.} \quad \sum\limits_{m\in M} v_i^m \leq 1, \forall i \in I \\
\qquad \sum\limits_{i\in I} x_i^k \leq 1, \forall k \in K \\
\qquad x_i^k \leq \sum\limits_{m\in M} v_i^m, \forall k \in K, i \in I. \\
\qquad \sum\limits_{j\in I} s_j^k x_j^k \geq s_i^k \sum\limits_{m\in M} v_i^m, \forall k \in K, i \in I \\
\qquad v_i^m, x_i^k \in \{0,1\}, \forall k \in K, i \in I, m \in M.
\end{cases}
$$

- Single-level reformulation of RPP, *[Domínguez, 2021]*.
- Solved with Gurobi.
- Time limit: 600 secs.

Comparative approaches

Naive approach

While a stopping criterion is not reached:

1) Randomly generate price vectors.
2) Solve the lower-level problems.
3) Evaluate the upper-level function.

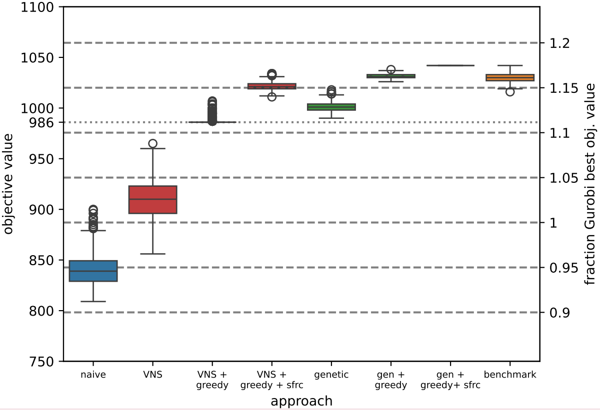
Output: keep the vector with the best objective value.

Comparative approaches

Benchmark

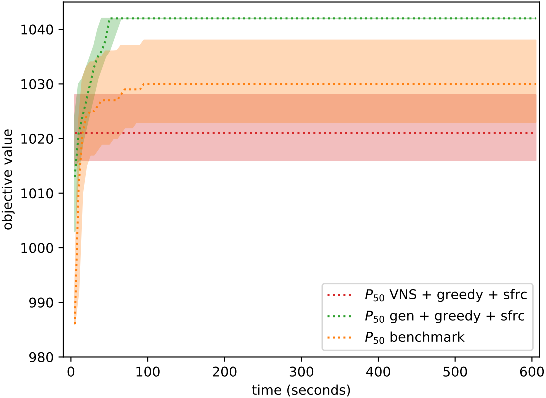
- Proposed in *Calvete et al. [2024]*.
- Greedy init. + genetic + optimization-based local search.

Instance $(K, I) = (30, 25)$



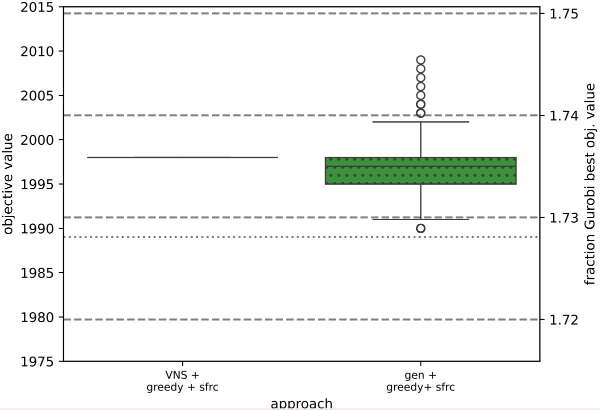
- Stopping criterion: 24000 points.
- Total solutions: $B^I = 23^{25} \approx 10^{34}$.
- Gurobi value in 600 seconds is 887.

Instance $(K, I) = (30, 25)$



- Stopping criterion: 24000 points.
- Total solutions: $B^I = 23^{25} \approx 10^{34}$.
- Gurobi value in 600 seconds is 887.

Instance $(K, I) = (60, 50)$



- Stopping criterion: 24000 points.
- Total solutions: $B^I = 40^{50} \approx 10^{80}$.
- Gurobi value in 600 seconds is 1151.

More details

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An enhanced heuristic framework for solving the Rank Pricing Problem

Asunción Jiménez-Cordero [a,b],*, Salvador Pineda [a,c], Juan Miguel Morales [a,b]

[a] Group OASYS, Ada Byron Research Building, Arquitecto Francisco Peñalosa Street, 18, Málaga, 29010, Spain
[b] Department of Mathematical Analysis, Statistics and Operations Research and Applied Mathematics (Area of Statistics and Operations Research), Universidad de Málaga, Málaga, Spain
[c] Department of Electric Engineering, Universidad de Málaga, Málaga, Spain

Available at:

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Conclusions

- RPP is a challenging combinatorial bilevel problem.
- Develop an heuristic approach for solving it.
- Conveniently define neighborhoods for escaping from local optima.
- Competitive with the comparative algorithms.
- Tested on several instances.

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- Competitive with the comparative algorithms.
- Tested on several instances.

Further research

- Combine the heuristic with other optimization methods.
- Handle more complex bilevel models.
- Integrate some machine learning strategies.

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`asuncionjc@uma.es`

JOINT WORK WITH:

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Thank you very much for for your attention!

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