The Cooperative Maximum Capture Facility Location Problem

SEIO 2023

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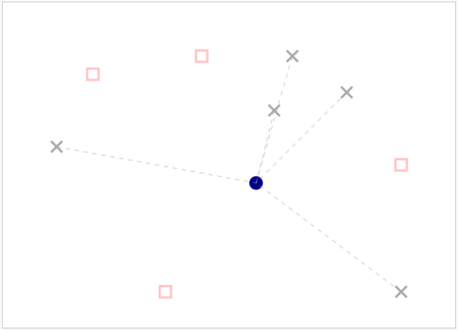
Introduction to the Cooperative Maximum Capture Facility Location (CMCFL)

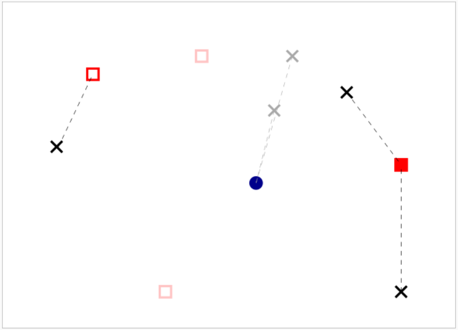
Maximum Capture Facility Location (MCFL)



Company?

Maximize customer capture by opening plants.

Maximum Capture Facility Location (MCFL)



Company?

Maximize customer capture by opening plants.

Customers?

Binary rule (Utility maximization): They patronize their **favorite open plant**

Cooperative MCFL. Applications?

Binary decision rule + cooperative setting

- **Cooperative capturing:** The captured utility is given in terms of an aggregation of the partial utilities.

- With **binary decision rule**: Customers are **loyal** to the company.

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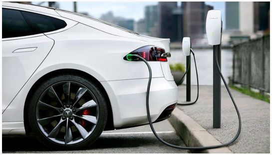

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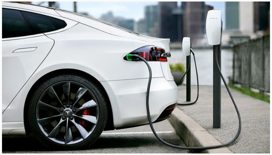


Membership for the service:

[1] EU deal to end sale of new CO2-emitting cars by 2035, Oct. 2023.

Modeling the captured utility

Captured utility

Ordered Median function[2]

Assigns importance weights to the sorted partial utilities, and then aggregates the weighted utilities.

Consider a set of customers $\mathcal{I}$, a set of plants $\mathcal{J}$, and **partial utilities** $\boldsymbol{u_{ij}^{ts}}$ for each time period t and scenario s. For $i \in \mathcal{I}$, define the associated weighting vector $\boldsymbol{\lambda_i} = (\lambda_{i1}, \ldots, \lambda_{i|\mathcal{J}|})$. Then the **captured utility** is:

$$U_i^{ts} := \Phi_{\boldsymbol{\lambda_i}}(u_{i1}^{ts}, \ldots, u_{i|\mathcal{J}|}^{ts}) = \sum_{j \in \mathcal{J}} \lambda_{ij} u_{i(j)}^{ts},$$

where $u_{i(r)}^{ts}$ is such that $u_{i(1)}^{ts} \geq \ldots \geq u_{i(|\mathcal{J}|)}^{ts}$.

$$\begin{cases} \lambda = (1,2,3) \\ u_i = (3,7,0) \end{cases} \quad \Rightarrow \quad U_i^{ts} = 1 \cdot 7 + 2 \cdot 3 + 3 \cdot 0 = 13.$$

[2] S. Nickel and J. Puerto. Location theory: a unified approach. *Springer Science & Business Media*, 2006.

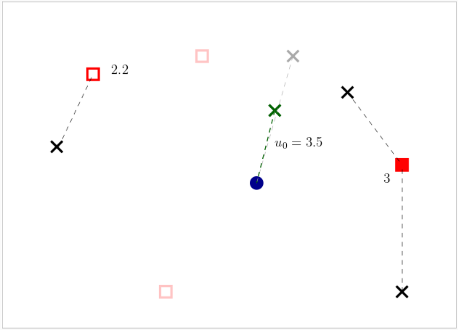
Unified cooperative framework

Different $\boldsymbol{\lambda}$ for different approaches:

Utility Maximization $\boldsymbol{\lambda_i} = (1, 0, \dots, 0)$ (non cooperative)

Consideration set of size ℓ $\boldsymbol{\lambda_i} = (\underbrace{1, \dots, 1}_{\ell}, 0, \dots, 0)$

Average $\boldsymbol{\lambda_i} = (\frac{1}{|\mathcal{J}|}, \dots, \frac{1}{|\mathcal{J}|})$



Cooperative setting with $\boldsymbol{\lambda_i} = (1, \frac{1}{2}, 0, \dots, 0)$:

Small plant: $U = 2.2$

Unified cooperative framework

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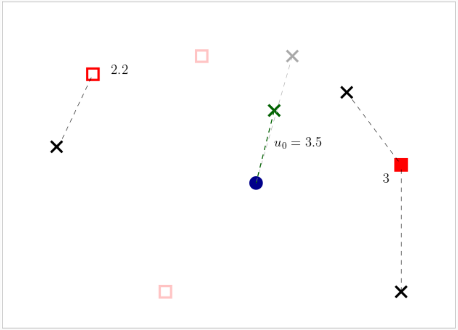

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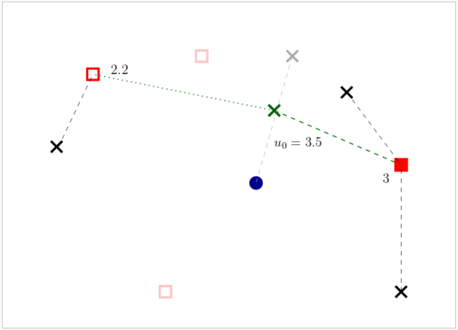

Cooperative setting with $\boldsymbol{\lambda_i} = (1, \frac{1}{2}, 0, \ldots, 0)$:

Small plant: $U = 2.2$

Big plant: $U = 3$

Both: $U = 3 \cdot 1 + \dfrac{2.2}{2} = 4.4$

Modeling the OMf

Variables

$$\sigma_{ijr}^{ts} := \begin{cases} 1, & u_{ij}^{ts} \text{ is the } r\text{-th largest utility for customer } i, \\ 0, & \text{otherwise.} \end{cases}$$

Modeling the OMf

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The **assignment problem** is:

$$\begin{aligned} U_i^{ts} = \quad & \max_{\boldsymbol{\sigma}_i^{ts}} \quad \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts} \\ & \text{s.t.} \quad \sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} = 1, \quad \forall r \in \mathcal{J}, \\ & \qquad \sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} = 1, \quad \forall j \in \mathcal{J}, \\ & \qquad \sum_{j \in \mathcal{J}} u_{ij}^{ts} \sigma_{ijr-1}^{ts} \geq \sum_{j \in \mathcal{J}} u_{ij}^{ts} \sigma_{ijr}^{ts}, \quad \forall r \in \mathcal{J} \setminus \{1\}, \\ & \qquad \sigma_{ijr}^{ts} \in \{0,1\}, \quad \forall j, r \in \mathcal{J}. \end{aligned}$$

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*Assuming a non-increasing vector $\boldsymbol{\lambda_i}$: $\lambda_{i1} \geq \cdots \geq \lambda_{i|\mathcal{J}|}$.

Bilevel Model for the CMCFL

Cooperative Maximum Capture Facility Location

Bilevel structure

First level Company deciding the locations of the plants:

$$x_{jk}^t := \begin{cases} 1, & \text{a facility of type } k \text{ is installed in } j \text{ at time period } t, \\ 0, & \text{otherwise.} \end{cases}$$

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Bilevel structure

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Second level Customers maximizing their utility:

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Partial utilities u_{ij}^{ts} (depend on the location variables x).

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Captured utility U_i^{ts} Embedded assignment problem in terms of σ, u.

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\max_{\boldsymbol{x},\boldsymbol{u}} \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{k'\in\mathcal{K}_j : k'\ge k} x_{jk'}^{t-1} \le \sum_{k'\in\mathcal{K}_j : k'\ge k} x_{jk'}^t, \qquad \forall j, t\in\mathcal{T}\setminus\{1\}, k, \\
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& u_{ij}^{ts} = \sum_{k\in\mathcal{K}_j} a_{ijk}^{ts} x_{jk}^t, \qquad \forall i, j, t, s, \\
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First approach: mixed-integer linear model

MIP

$$
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& \sigma_{ijr}^{ts}, z_i^{ts} \in [0, 1], \quad \forall i, t, s, j, r.
\end{aligned}
$$

[3, 4, 5, 1, 2]

Linearization I

1. Apply a perspective transformation[3]. z remains continuous

$$
\begin{aligned}
& u_{i0}^{ts} z_i^{ts} \le \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} \color{red}{u_{ij}^{ts} \sigma_{ijr}^{ts} z_i^{ts}}, \quad \forall i, t, s, \\
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[3, 4, 5, 1, 2]

[3] Günlük, O., Linderoth, J., 2012. Perspective Reformulation and Applications, in: Lee, J., Leyffer, S. (Eds.), Mixed Integer Nonlinear Programming, Springer, New York, NY. pp. 61–89.

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\begin{aligned}
& u_{i0}^{ts} z_i^{ts} \leq \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts}, \quad \forall i, t, s, \\
& \sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, r, \\
& \sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, j, \\
& \sigma_{ijr}^{ts}, z_i^{ts} \in [0, 1], \quad \forall i, t, s, j, r.
\end{aligned}
$$

[3, 4, 5, 1, 2]

[3] Günlük, O., Linderoth, J., 2012. Perspective Reformulation and Applications, in: Lee, J., Leyffer, S. (Eds.), Mixed Integer Nonlinear Programming, Springer, New York, NY. pp. 61–89.

Linearization I

1. Apply a perspective transformation[3]. z remains continuous

$$
\begin{aligned}
& u_{i0}^{ts} z_i^{ts} \leq \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts}, \quad \forall i, t, s, \\
& \sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, r, \\
& \sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, j, \\
& \sigma_{ijr}^{ts}, z_i^{ts} \in [0, 1], \quad \forall i, t, s, j, r.
\end{aligned}
$$

[3, 4, 5, 1, 2]

[3] Günlük, O., Linderoth, J., 2012. Perspective Reformulation and Applications, in: Lee, J., Leyffer, S. (Eds.), Mixed Integer Nonlinear Programming, Springer, New York, NY. pp. 61–89.

Linearization II

Standard linearization of bilinear terms with binary variables. σ needs to be integer:

Define new auxiliary variables $w_{ijr}^{ts} := u_{ij}^{ts}\sigma_{ijr}^{ts}$

$$w_{ijr}^{ts} := \begin{cases} u_{ij}^{ts} = \sum_{k\in\mathcal{K}_j} a_{ijk}^{ts}x_{jk}^{t}, & u_{ij}^{ts} \text{ is the } r\text{-th largest utility for customer } i, \\ & 0 = a_{ij0}^{ts} \leq a_{ij1}^{ts} \leq \cdots \leq a_{ij|\mathcal{K}_j|}^{ts} \\ 0, & \text{otherwise.} \end{cases}$$

Linearization II

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Use constraints:

$$w_{ijr}^{ts} \leq M_{ij}^{ts}\sigma_{ijr}^{ts}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}, j, r \in \mathcal{J},$$
$$w_{ijr}^{ts} \leq u_{ij}^{ts}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}, j, r \in \mathcal{J},$$

Valid inequalities and preprocessing

$w_{ijr}^{ts} := u_{ij}^{ts}\sigma_{ijr}^{ts}$

$$w_{ijr}^{ts} := \begin{cases} u_{ij}^{ts} = \sum_{k\in\mathcal{K}_j} a_{ijk}^{ts} x_{jk}^{t}, & u_{ij}^{ts} \text{ is the } r\text{-th largest utility for customer } i, \\ & 0 = a_{ij0}^{ts} \le a_{ij1}^{ts} \le \cdots \le a_{ij|\mathcal{K}_j|}^{ts} \\ 0, & \text{otherwise.} \end{cases}$$

Valid inequalities and preprocessing

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$$w_{ijr}^{ts} \le a_{ijk}^{ts}\sigma_{ijr}^{ts} + \sum_{\substack{k'\in\mathcal{K}_j:\\ k'>k}} (a_{ijk'}^{ts} - a_{ijk}^{ts}) x_{jk'}^{t}, \qquad \forall j, r, k,$$

Valid inequalities and preprocessing

$w_{ijr}^{ts} := u_{ij}^{ts}\sigma_{ijr}^{ts}$

$$w_{ijr}^{ts} := \begin{cases} u_{ij}^{ts} = \sum_{k \in \mathcal{K}_j} a_{ijk}^{ts} x_{jk}^{t}, & u_{ij}^{ts} \text{ is the } r\text{-th largest utility for customer } i, \\ & 0 = a_{ij0}^{ts} \le a_{ij1}^{ts} \le \cdots \le a_{ij|\mathcal{K}_j|}^{ts} \\ 0, & \text{otherwise.} \end{cases}$$

$$w_{ijr}^{ts} \le a_{ijk}^{ts}\sigma_{ijr}^{ts} + \sum_{\substack{k' \in \mathcal{K}_j: \\ k' > k}} (a_{ijk'}^{ts} - a_{ijk}^{ts}) x_{jk'}^{t}, \qquad \forall j, r, k,$$

$$\sum_{r \in \mathcal{J}} w_{ijr}^{ts} \le \sum_{r \in \mathcal{J}} a_{ijk}^{ts}\sigma_{ijr}^{ts} + \sum_{\substack{k' \in \mathcal{K}_j: \\ k' > k}} (a_{ijk'}^{ts} - a_{ijk}^{ts}) x_{jk'}^{t}, \qquad \forall j, k,$$

Valid inequalities and preprocessing

$w^{ts}_{ijr} := u^{ts}_{ij}\sigma^{ts}_{ijr}$

$$w^{ts}_{ijr} := \begin{cases} u^{ts}_{ij} = \sum_{k\in\mathcal{K}_j} a^{ts}_{ijk}x^t_{jk}, & u^{ts}_{ij} \text{ is the } r\text{-th largest utility for customer } i, \\ & 0 = a^{ts}_{ij0} \le a^{ts}_{ij1} \le \cdots \le a^{ts}_{ij|\mathcal{K}_j|} \\ 0, & \text{otherwise.} \end{cases}$$

$$w^{ts}_{ijr} \le a^{ts}_{ijk}\sigma^{ts}_{ijr} + \sum_{\substack{k'\in\mathcal{K}_j:\\ k'>k}} (a^{ts}_{ijk'} - a^{ts}_{ijk})x^t_{jk'}, \qquad \forall j, r, k,$$

$$\sum_{r\in\mathcal{J}} w^{ts}_{ijr} \le \sum_{r\in\mathcal{J}} a^{ts}_{ijk}\sigma^{ts}_{ijr} + \sum_{\substack{k'\in\mathcal{K}_j:\\ k'>k}} (a^{ts}_{ijk'} - a^{ts}_{ijk})x^t_{jk'}, \qquad \forall j, k,$$

$$\sum_{r\in\mathcal{J}} \sigma^{ts}_{ijr} \le \sum_{k\in\mathcal{K}_j} x^t_{jk}, \qquad \forall j,$$

$$\sigma^{ts}_{ijr} = 0 \qquad \forall i, t, s, j, r : \lambda_{ir} = 0.$$

Second approach: Benders' decomposition

Master Problem

$$
\begin{aligned}
\max_{x,z} \quad & \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|} \sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \leq 1, \qquad \forall j, t, \\
& \sum_{\substack{k'\in\mathcal{K}_j: \\ k'\geq k}} x_{jk'}^{t-1} \leq \sum_{\substack{k'\in\mathcal{K}_j: \\ k'\geq k}} x_{jk'}^t, \qquad \forall j, t \in \mathcal{T}\setminus\{1\}, k, \\
& \sum_{\substack{t'\in\mathcal{T}: \\ t'\leq t}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} c_{jk}^{t'} (x_{jk}^{t'} - x_{jk}^{t'-1}) \leq \sum_{\substack{t'\in\mathcal{T}: \\ t'\leq t}} b^{t'}, \qquad \forall t, \\
& x_{jk}^t \in \{0,1\}, \qquad \forall j, t, k, \\
& {\color{red} u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(x) z_i^{ts}}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}.
\end{aligned}
$$

Master Problem

$$
\begin{aligned}
\max_{\boldsymbol{x},\boldsymbol{z}} \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{\substack{k'\in\mathcal{K}_j:\\ k'\ge k}} x_{jk'}^{t-1} \le \sum_{\substack{k'\in\mathcal{K}_j:\\ k'\ge k}} x_{jk'}^{t}, \qquad \forall j, t\in\mathcal{T}\setminus\{1\}, k, \\
& \sum_{\substack{t'\in\mathcal{T}:\\ t'\le t}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_j} c_{jk}^{t'}(x_{jk}^{t'} - x_{jk}^{t'-1}) \le \sum_{\substack{t'\in\mathcal{T}:\\ t'\le t}} b^{t'}, \qquad \forall t, \\
& x_{jk}^t \in \{0,1\}, \qquad \forall j, t, k, \\
& u_{0i}^{ts} z_i^{ts} \le U_i^{ts}(x) z_i^{ts} \le U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_j}\left[\bar{s}_{jk}(x_{jk}^t - \bar{x}_{jk}^t)\right]^+, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, s\in\mathcal{S}.
\end{aligned}
$$

Subproblems

$$u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \left[\bar{s}_{jk}(x_{jk}^t - \bar{x}_{jk}^t)\right]^+$$

$$
\begin{aligned}
(\text{PR}) \quad \max_{\sigma} \quad & \sum_{j\in\mathcal{J}} \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \lambda_r a_{jk} \sigma_{jkr} \\
\text{s.t.} \quad & \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall r, \\
& \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall j, \\
& \sum_{r\in\mathcal{J}} \sigma_{jkr} \leq \bar{x}_{jk}, \quad \forall j, k \\
& \sigma_{jkr} \geq 0, \quad \forall j, r, k.
\end{aligned}
$$

Subproblems

$$u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \left[\bar{s}_{jk} (x_{jk}^t - \bar{x}_{jk}^t) \right]^+$$

$$\begin{aligned}
\text{(PR)} \quad \max_{\sigma} \quad & \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \lambda_r a_{jk} \sigma_{jkr} \\
\text{s.t.} \quad & \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall r, \quad (\gamma_r) \\
& \sum_{r \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall j, \quad (\delta_j) \\
& \sum_{r \in \mathcal{J}} \sigma_{jkr} \leq \bar{x}_{jk}, \quad \forall j, k \quad (\eta_{jk}) \\
& \sigma_{jkr} \geq 0, \quad \forall j, r, k.
\end{aligned}$$

$$\begin{aligned}
\text{(DU)} \quad \min \quad & \sum_{r \in \mathcal{J}} \gamma_r + \sum_{j \in \mathcal{J}} \delta_j + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \bar{x}_{jk} \eta_{jk} \\
\text{s.t.} \quad & \gamma_r + \delta_j + \eta_{jk} \geq \lambda_r a_{jk}, \quad \forall j, r \in \mathcal{J}, k \in \mathcal{K}_j.
\end{aligned}$$

Subproblems

$$u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \left[\bar{s}_{jk}(x_{jk}^t - \bar{x}_{jk}^t) \right]^+$$

$$\begin{aligned}
(\text{PR}) \quad \max_{\sigma} \quad & \sum_{j\in\mathcal{J}} \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \lambda_r a_{jk} \sigma_{jkr} \\
\text{s.t.} \quad & \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall r, \\
& \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall j, \\
& \sum_{r\in\mathcal{J}} \sigma_{jkr} \leq \bar{x}_{jk}, \quad \forall j, k \\
& \sigma_{jkr} \geq 0, \quad \forall j, r, k.
\end{aligned}$$

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(\text{DU}) \quad \min \quad & \sum_{r\in\mathcal{J}} \gamma_r + \sum_{j\in\mathcal{J}} \delta_j + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \bar{x}_{jk} \eta_{jk} \\
\text{s.t.} \quad & \gamma_r + \delta_j + \eta_{jk} \geq \lambda_r a_{jk}, \quad \forall j, r \in \mathcal{J}, k \in \mathcal{K}_j.
\end{aligned}$$

Subproblems

$u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \left[\bar{s}_{jk}(x_{jk}^t - \bar{x}_{jk}^t)\right]^+$

$$
\begin{aligned}
\text{(PR)} \quad \max_{\sigma} \quad & \sum_{j\in\mathcal{J}} \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \lambda_r a_{jk} \sigma_{jkr} \\
\text{s.t.} \quad & \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall r, \\
& \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall j, \\
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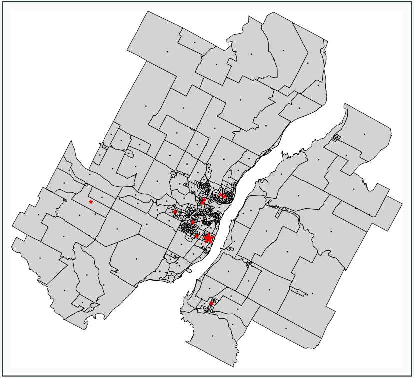
& \sigma_{jkr} \geq 0, \quad \forall j, r, k.
\end{aligned}
$$

$$
\begin{aligned}
\text{(DU)} \quad \min \quad & \sum_{r\in\mathcal{J}} \gamma_r + \sum_{j\in\mathcal{J}} \delta_j + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \bar{x}_{jk} \eta_{jk} \\
\text{s.t.} \quad & \gamma_r + \delta_j + \eta_{jk} \geq \lambda_r a_{jk}, \quad \forall j, r \in \mathcal{J}, k \in \mathcal{K}_j.
\end{aligned}
$$

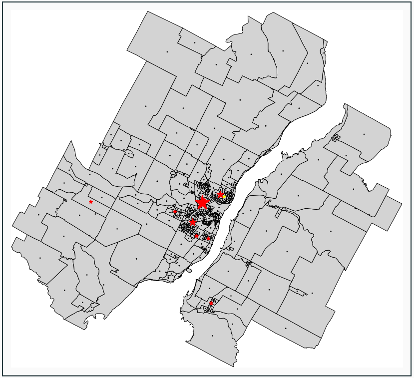
Tailored Primal-Dual algorithm to solve (PR)-(DU) at each iteration.

Computational Experiments

Case Study: Electric Vehicle Charging Stations in Trois-Rivières, Canada[4]



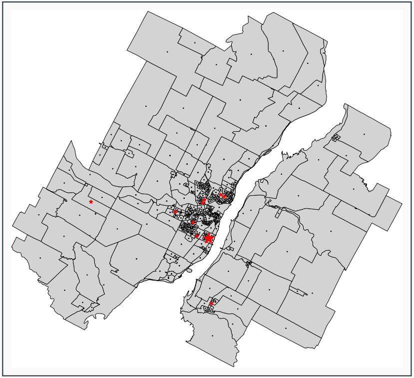
(a) $\boldsymbol{\lambda} = C = (1, 0, \ldots, 0)$



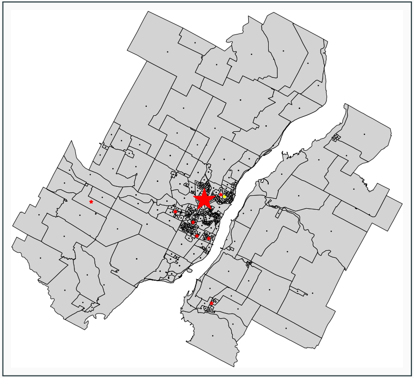
(b) $\boldsymbol{\lambda} = K = (1, 1, 0, \ldots, 0)$

[4] S. Lamontagne et al. Optimising electric vehicle charging station placement using advanced discrete choice models. *INFORMS Journal on Computing*, 2023.

Case Study: Electric Vehicle Charging Stations in Trois-Rivières, Canada[4]



(a) $\boldsymbol{\lambda} = C = (1, 0, \dots, 0)$



(c) $\boldsymbol{\lambda} = L = (1, \frac{1}{2}, 0, \dots, 0)$

[4] S. Lamontagne et al. Optimising electric vehicle charging station placement using advanced discrete choice models. *INFORMS Journal on Computing*, 2023.

Conclusions and future work

Conclusions

The CMCFL introduces a **cooperative framework** for the MCFL with binary choice rule.

Future work

- Add **capacities** to the facilities dependent on the type of plant installed $\Rightarrow$ Bilevel location-allocation problem.
- Robustify the OMf associated to the utility. For instance considering **non-monotone or negative λ-weights**, or **variable λ-weights** that meet certain conditions associated to the knowledge of the customer, and optimize U in the worst-case.

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Thank you for your attention!

Questions?

C. Domínguez, R. Gázquez, J. M. Morales, and S. Pineda. **The cooperative maximum capture facility location problem.** *arXiv preprint arXiv:2305.15169*, 2023.