# Tightening big-M for Optimal Transmission Switching

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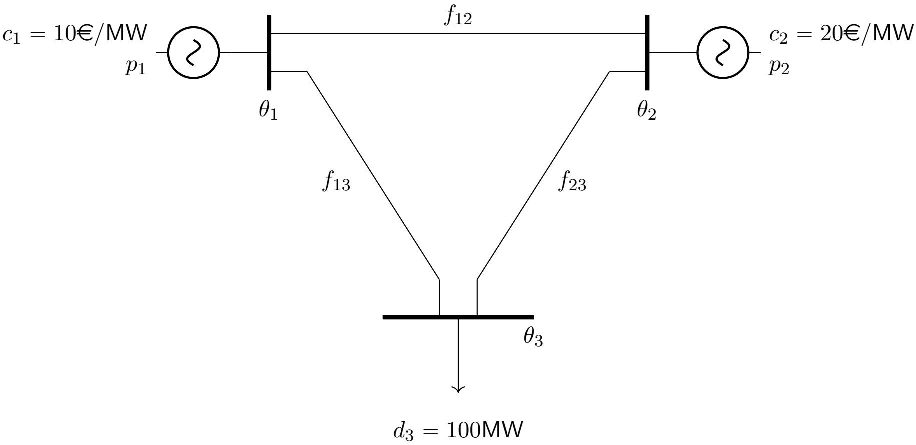
Salvador Pineda

(joint work with J. M. Morales, C. Dominguez, A. Porras)

OASYS group, University of Málaga (Spain)

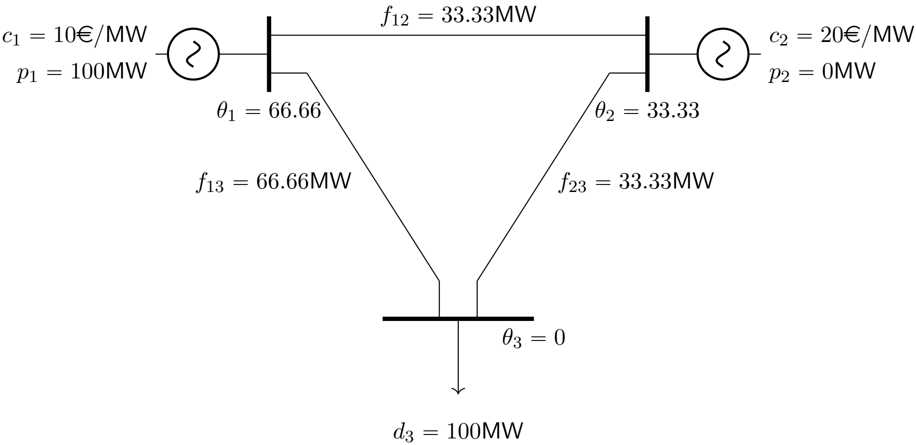
# Motivating example

Optimal power flow (OPF): Determine the power generation and power flows to satisfy the demand at the minimum cost



# Motivating example

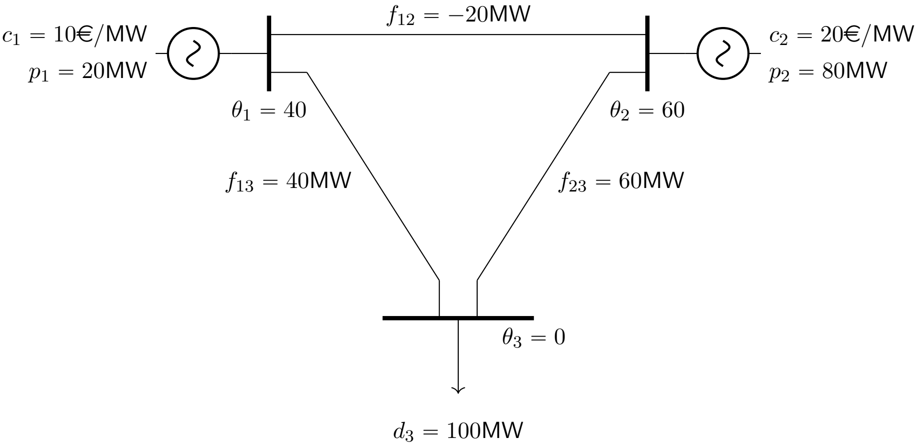
Optimal solution: generate all with cheapest unit (cost = 1000€)




Electrons are not potatoes!!!

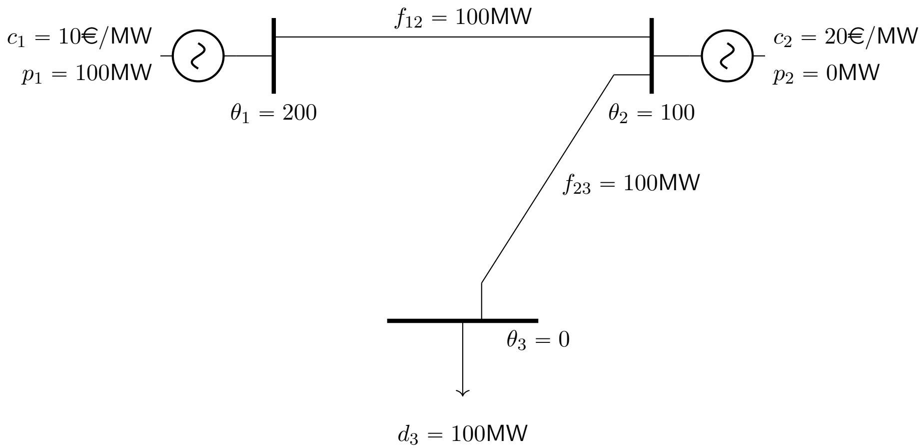
# Motivating example

If $f_{13} \leqslant 40$, the expensive unit also generates (cost=1800€)



# Motivating example

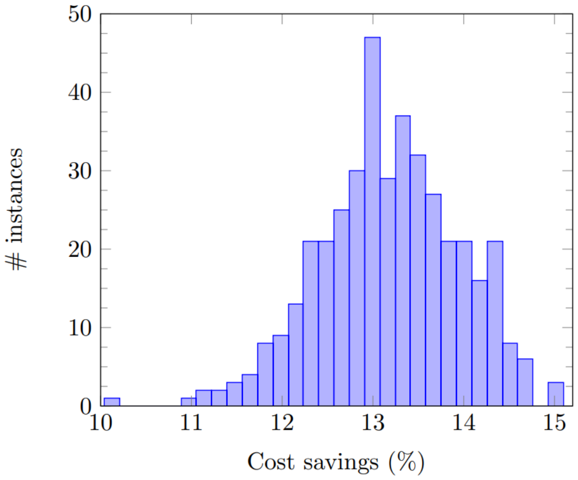
If line $13$ is disconnected, cost$=$ 1000€




Disconnecting lines can reduce cost!!!

# Motivating example

In the 118-bus system, the average cost saving is 13.2%



# Methodology

The optimal power flow (OPF) is formulated as a linear optimization problem

$$
\begin{aligned}
\min_{p_i, f_{ij}, \theta_i} \quad & \sum_i c_i \, p_i && \text{(1a)} \\
\text{s.t.} \quad & \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(1b)} \\
& f_{ij} = b_{ij}(\theta_i - \theta_j), \quad \forall (i,j) \in \mathcal{L} && \text{(1c)} \\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(1d)} \\
& -\underline{f}_{ij} \leqslant f_{ij} \leqslant \overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(1e)}
\end{aligned}
$$

# Methodology

The optimal transmission switching (OTS) requires binary variables $x_{ij}$ and is formulated as a mixed-integer non-linear problem ...

$$
\begin{aligned}
\min_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & \sum_i c_i \, p_i && \text{(2a)} \\
\text{s.t.} \quad & \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(2b)} \\
& f_{ij} = x_{ij} b_{ij} (\theta_i - \theta_j), \quad \forall (i,j) \in \mathcal{L} && \text{(2c)} \\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(2d)} \\
& - x_{ij} \underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij} \overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(2e)} \\
& x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in \mathcal{L} && \text{(2f)}
\end{aligned}
$$

... that can be directly solved using optimization solvers

# Methodology

To avoid the non-linear terms in

$$f_{ij} = x_{ij} b_{ij}(\theta_i - \theta_j)$$

We replace it by

$$b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij})$$

Together with equation

$$-x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}$$

We have that:

- If $x_{ij} = 1 \Rightarrow b_{ij}(\theta_i - \theta_j) \leqslant f_{ij} \leqslant b_{ij}(\theta_i - \theta_j)$ and $-\underline{f}_{ij} \leqslant f_{ij} \leqslant \overline{f}_{ij}$

- If $x_{ij} = 0 \Rightarrow f_{ij} = 0$ and $\underline{M}_{ij} \leqslant b_{ij}(\theta_i - \theta_j) \leqslant \overline{M}_{ij}$

# Methodology

The OTS is reformulated as a mixed-integer linear problem

$$
\begin{aligned}
\min_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & \sum_i c_i \, p_i && \text{(3a)} \\
\text{s.t.} \quad & \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(3b)} \\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(3c)} \\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(3d)} \\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(3e)} \\
& -x_{ij} \underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij} \overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(3f)} \\
& x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in \mathcal{L} && \text{(3g)}
\end{aligned}
$$

$\underline{M}_{ij}$ and $\overline{M}_{ij}$ must be large enough to be valid bounds of $b_{ij}(\theta_i - \theta_j)$

$\underline{M}_{ij}$ and $\overline{M}_{ij}$ must be small enough to avoid computational issues

# Methodology

Since $\underline{M}_{ij} \leqslant b_{ij}(\theta_i - \theta_j) \leqslant \overline{M}_{ij}$ when $x_{ij} = 0$, we can compute these bounds for a particular switchable line $(i', j')$ as

$$
\begin{aligned}
\overline{M}^{\mathrm{OPT}}_{i'j'} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & b_{i'j'}(\theta_{i'} - \theta_{j'}) && \text{(4a)}\\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(4b)}\\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(4c)}\\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(4d)}\\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(4e)}\\
& -x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(4f)}\\
& x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in \mathcal{L} && \text{(4g)}\\
& x_{i'j'} = 0 && \text{(4h)}
\end{aligned}
$$

However, this problem is as difficult as the original one!!

# Methodology

Fattahi et. al (2019) find a bound on $\overline{M}_{ij}^{\mathrm{OPT}}$ if there exists a connected spanning subgraph of the network with non-switchable lines

$$\overline{M}_{i'j'}^{\mathrm{OPT}} \geqslant b_{i'j'} \sum_{(k,l) \in SP_{i'j'}} \frac{\overline{f}_{kl}}{b_{kl}}$$

where $SP_{i'j'}$ is the shortest path between nodes $i'$ and $j'$ (very easy to compute using Dijkstra's algorithm)

# Methodology

Instead of solving the original bounding problem to compute $\overline{M}_{ij}^{\text{opt}}$, we solve the following linear relaxation

$$
\begin{aligned}
\overline{M}_{i'j'}^{\text{LR}} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & b_{i'j'}(\theta_{i'} - \theta_{j'}) && \text{(5a)}\\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(5b)}\\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(5c)}\\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(5d)}\\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(5e)}\\
& -x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(5f)}\\
& 0 \leqslant x_{ij} \leqslant 1, \quad \forall (i,j) \in \mathcal{L} && \text{(5g)}\\
& x_{i'j'} = 0 && \text{(5h)}
\end{aligned}
$$

This problem can be "too relaxed" and provide poor bounds...

# Methodology

Inspired by Porras et. at (2022), we include a bound on the cost using a "reasonable good" feasible solution.

$$\sum_i c_i p_i \leqslant \text{cost}$$

A **naive approach** is to satisfy the demand with the most expensive units. We denote this cost as $\text{cost}^{\text{NAI}}$

Another option is using **greedy heuristics**. For instance, we can start with all lines connected and disconnect the line that involves highest savings by solving several OPF problem. We repeat the process until any disconnection increases the cost. We denote this as $\text{cost}^{\text{HEU}}$

# Methodology

If the bound on the cost is considered, we solve the following problem

$$
\begin{aligned}
\overline{M}^{\mathrm{LR}}_{i'j'} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & b_{i'j'}(\theta_{i'} - \theta_{j'}) && \text{(6a)} \\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(6b)} \\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(6c)} \\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(6d)} \\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(6e)} \\
& - x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(6f)} \\
& 0 \leqslant x_{ij} \leqslant 1, \quad \forall (i,j) \in \mathcal{L} && \text{(6g)} \\
& x_{i'j'} = 0 && \text{(6h)} \\
& \sum_i c_i p_i \leqslant \text{cost}^{\mathrm{NAI/HEU}} && \text{(6i)}
\end{aligned}
$$

We also solve bounding problems to improve $\underline{f}_{ij}$ and $\overline{f}_{ij}$ as follows

$$
\begin{aligned}
\overline{f}^{\mathrm{LR}}_{i'j'} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} \quad & b_{i'j'}(\theta_{i'} - \theta_{j'}) && \text{(7a)}\\
\text{s.t.} \quad & \sum_{(i,j)\in\mathcal{L}_i^-} f_{ij} - \sum_{(i,j)\in\mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i && \text{(7b)}\\
& b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leqslant f_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(7c)}\\
& f_{ij} \leqslant b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i,j) \in \mathcal{L} && \text{(7d)}\\
& \underline{p}_i \leqslant p_i \leqslant \overline{p}_i, \quad \forall i && \text{(7e)}\\
& -x_{ij}\underline{f}_{ij} \leqslant f_{ij} \leqslant x_{ij}\overline{f}_{ij}, \quad \forall (i,j) \in \mathcal{L} && \text{(7f)}\\
& 0 \leqslant x_{ij} \leqslant 1, \quad \forall (i,j) \in \mathcal{L} && \text{(7g)}\\
& x_{i'j'} = 1 && \text{(7h)}\\
& \sum_i c_i p_i \leqslant \text{cost}^{\mathrm{NAI/HEU}} && \text{(7i)}
\end{aligned}
$$

# Methodology

The proposed method runs as follows:

1. Set $\underline{f}_{ij}$, $\overline{f}_{ij}$ to original capacities
2. Compute $\underline{M}_{ij}$, $\overline{M}_{ij}$ using Fattahi's method
3. Compute the cost bound cost$^{\mathrm{NAI}}$ or cost$^{\mathrm{HEU}}$
4. Solve bounding problems (LP) to adjust $\underline{f}_{ij}$, $\overline{f}_{ij}, \underline{M}_{ij}$, $\overline{M}_{ij}$
5. Repeat step 4 if needed
6. Solve the OTS problem with improved bounds and maximum cost

# Numerical simulations

- 118-bus system with 186-lines
- 100 instances with different demands and connected subgraphs
- Each instance includes 69 switchable lines
- Total time = bounding problems (LPs) and final OTS problem (MIP)
- GAP at 0.01% and maximum time 1 hour

# Numerical simulations

- 1 round of bounding problems
- $\Delta^M$: Relative bigM values with respect to Fattahi
- $\Delta^L$: Relative line capacities with respect to original

| Method | $\Delta^M$ | $\Delta^L$ | Time (s) | Unsolved | Max gap |
|---|---|---|---|---|---|
| Fattahi | 100% | 100% | 672 | 14 | 0.69% |
| $\text{cost}^{\text{NAI}}$ | 68% | 74% | 487 | 8 | 1.04% |
| $\text{cost}^{\text{HEU}}$ | 64% | 68% | 170 | 1 | 0.03% |

- Even without the cost constraint, the bounding problems improve computational performance
- Using the cost of a feasible solution close to the optimal further improves the performance of the proposed methodology

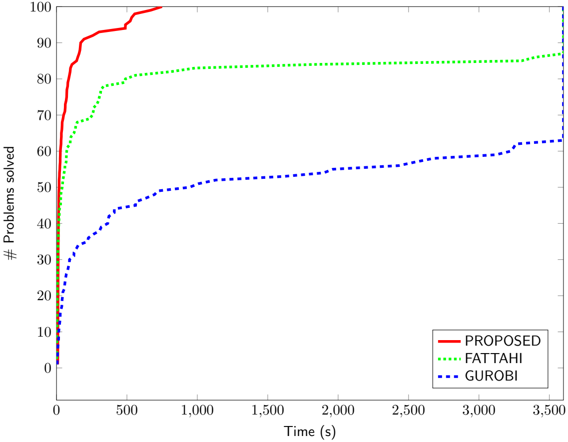
# Numerical simulations

- Upper bound cost$^{\mathrm{HEU}}$
- More rounds of bounding problems

| Method | $\Delta^M$ | $\Delta^L$ | Time (s) | Unsolved | Max gap |
|---|---|---|---|---|---|
| Fattahi | 100% | 100% | 672 | 14 | 0.69% |
| 1 round | 64% | 68% | 170 | 1 | 0.03% |
| 2 rounds | 56% | 63% | 87 | 0 | - |
| 3 rounds | 52% | 61% | 81 | 0 | - |
| 4 rounds | 50% | 59% | 80 | 0 | - |

- The bounds $\underline{f}_{ij}$, $\overline{f}_{ij}$, $\underline{M}_{ij}$, $\overline{M}_{ij}$ are tightened in each iteration
- With four rounds the average time is reduced by 88%

# Numerical simulations




- General-purpose solvers perform poorly (38 unsolved problems)
- Fattahi uses knowledge about the problem to adjust bigMs
- Our bound tightening method is the winner (max time 13 minutes)

# Conclusions

- The optimal transmission switching (OTS) determines the lines that can be disconnected to reduce the operating cost.
- The OTS is formulated as a mixed-integer linear problem with bigMs that is computationally difficult to solve.
- We propose a methodology that solves relaxed bounding problems to find tight bigMs and reduce the computational burden of the OTS.

# Thanks for the attention!! Questions??

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## Tight Big-Ms for Optimal Transmission Switching

Salvador Pineda, Juan Miguel Morales, Álvaro Porras, Concepción Domínguez
OASYS research group, University of Málaga, Spain
{spineda, juan.morales, alvaroporras, concepcion.dominguez}@uma.es

OASYS
Optimization and Analytics for Sustainable energY Systems

More info: oasys.uma.es
Email: spineda@uma.es