Bilevel formulation and two exact resolution approaches for the Maximum Capture Facility Location with Random OWA utilities

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ICH Instituto Chileno de Investigación

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ISCI Instituto Sistemas Complejos de Ingeniería

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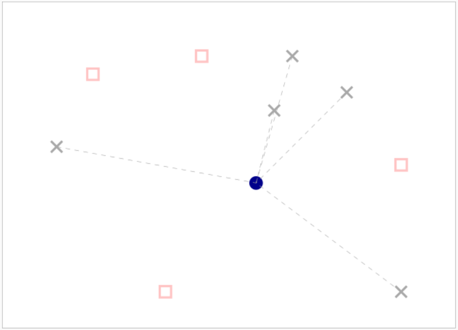
Introduction to the Cooperative Maximum Capture Facility Location (CMCFL)

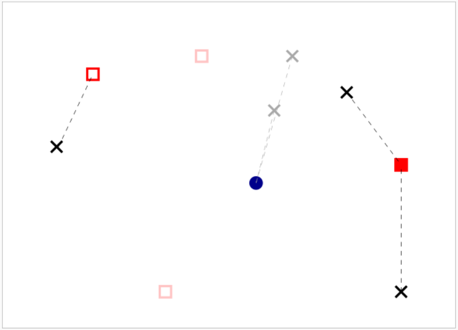
Maximum Capture Facility Location (MCFL)



Company?

Maximize customer capture by opening plants.

Maximum Capture Facility Location (MCFL)



Company?

Maximize customer capture by opening plants.

Customers?

Binary rule (Utility maximization): They patronize their **favorite open plant**

Cooperative MCFL. Applications?

Binary decision rule + cooperative setting

- **Cooperative capturing:** The captured utility is given in terms of an aggregation of the partial utilities.

- With **binary decision rule**: Customers are **loyal** to the company.

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Specific product:

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Specific product:

Membership for the service:

Modeling the captured utility

Captured utility

Ordered Median function

Assigns importance weights to the sorted partial utilities, and then aggregates the weighted utilities.

Consider a set of customers $\mathcal{I}$, a set of plants $\mathcal{J}$, and **partial utilities** $\boldsymbol{u_{ij}^{ts}}$ for each time period t and scenario s. For $i \in \mathcal{I}$, define the associated weighting vector $\boldsymbol{\lambda_i} = (\lambda_{i1}, \ldots, \lambda_{i|\mathcal{J}|})$. Then the **captured utility** is:

$$U_i^{ts} := \Phi_{\boldsymbol{\lambda_i}}(u_{i1}^{ts}, \ldots, u_{i|\mathcal{J}|}^{ts}) = \sum_{j \in \mathcal{J}} \lambda_{ij} u_{i(j)}^{ts},$$

where $u_{i(r)}^{ts}$ is such that $u_{i(1)}^{ts} \geq \ldots \geq u_{i(|\mathcal{J}|)}^{ts}$.

$$\begin{cases} \lambda = (1,2,3) \\ u_i = (3,7,0) \end{cases} \quad \Rightarrow \quad U_i^{ts} = 1 \cdot 7 + 2 \cdot 3 + 3 \cdot 0 = 13.$$

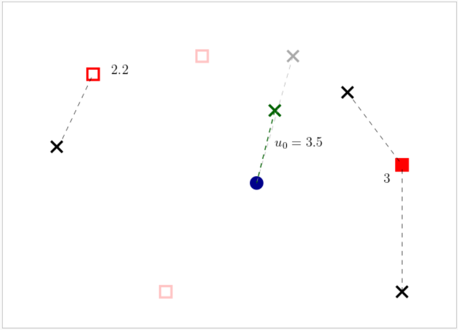
Unified cooperative framework

Different $\boldsymbol{\lambda}$ for different approaches:

Utility Maximization $\boldsymbol{\lambda_i} = (1, 0, \ldots, 0)$ (non cooperative)

Consideration set of size ℓ $\boldsymbol{\lambda_i} = (\underbrace{1, \ldots, 1}_{\ell}, 0, \ldots, 0)$

Average $\boldsymbol{\lambda_i} = (\frac{1}{|\mathcal{J}|}, \ldots, \frac{1}{|\mathcal{J}|})$



Cooperative setting with $\boldsymbol{\lambda_i} = (1, \frac{1}{2}, 0, \ldots, 0)$:

Small plant: $U = 2.2$

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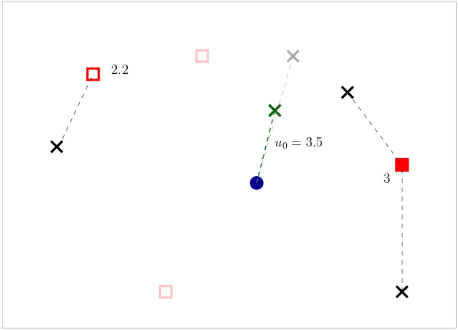

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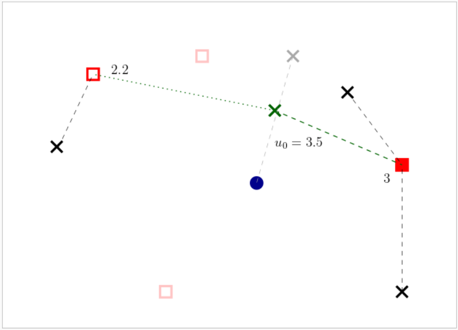

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Small plant: $U = 2.2$

Big plant: $U = 3$

Both: $U = 3 \cdot 1 + \dfrac{2.2}{2} = 4.4$

Modeling the OMf

Variables

$$\sigma_{ijr}^{ts} := \begin{cases} 1, & u_{ij}^{ts} \text{ is the } r\text{-th largest utility for customer } i, \\ 0, & \text{otherwise.} \end{cases}$$

Modeling the OMf

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The **assignment problem** is:

$$\begin{aligned} U_i^{ts} = \quad & \max_{\boldsymbol{\sigma}_i^{ts}} && \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts} \\ & \text{s.t.} && \sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} = 1, \quad \forall r \in \mathcal{J}, \\ &&& \sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} = 1, \quad \forall j \in \mathcal{J}, \\ &&& \sum_{j \in \mathcal{J}} u_{ij}^{ts} \sigma_{ijr-1}^{ts} \geq \sum_{j \in \mathcal{J}} u_{ij}^{ts} \sigma_{ijr}^{ts}, \quad \forall r \in \mathcal{J} \setminus \{1\}, \\ &&& \sigma_{ijr}^{ts} \in \{0, 1\}, \quad \forall j, r \in \mathcal{J}. \end{aligned}$$

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*Assuming a non-increasing vector $\boldsymbol{\lambda_i}$: $\lambda_{i1} \geq \cdots \geq \lambda_{i|\mathcal{J}|}$.

Bilevel Model for the CMCFL

Cooperative Maximum Capture Facility Location

Bilevel structure

First level Company deciding the locations of the plants:

$$x_{jk}^{t} := \begin{cases} 1, & \text{a facility of type } k \text{ is installed in } j \text{ at time period } t, \\ 0, & \text{otherwise.} \end{cases}$$

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Partial utilities u_{ij}^{ts} (depend on the location variables x).

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Captured utility U_{i}^{ts} Embedded assignment problem in terms of σ, u.

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\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{k'\in\mathcal{K}_j : k' \ge k} x_{jk'}^{t-1} \le \sum_{k'\in\mathcal{K}_j : k' \ge k} x_{jk'}^t, \qquad \forall j, t \in \mathcal{T}\setminus\{1\}, k, \\
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First approach: mixed-integer linear model

Single-level MIP formulation

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& \qquad\qquad\qquad \sum_{r\in\mathcal{J}} \sigma_{ijr}^{ts} \le 1, \qquad \forall i,t,s,j, \\
& \qquad\qquad\qquad \sigma_{ijr}^{ts} \in [0,1], \qquad \forall i,t,s,j,r.
\end{aligned}
$$

Single-level MIP formulation

$$
\begin{aligned}
\max \quad & \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k \in \mathcal{K}_j} x_{jk}^t \leq 1, \qquad \forall j, t, \\
& \sum_{k' \in \mathcal{K}_j : k' \geq k} x_{jk'}^{t-1} \leq \sum_{k' \in \mathcal{K}_j : k' \geq k} x_{jk'}^t, \qquad \forall j, t \in \mathcal{T} \setminus \{1\}, k, \\
& \sum_{t' \in \mathcal{T} : t' \leq t} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} c_{jk}^{t'} (x_{jk}^{t'} - x_{jk}^{t'-1}) \leq \sum_{t' \in \mathcal{T} : t' \leq t} b^{t'}, \qquad \forall t, \\
& u_{ij}^{ts} = \sum_{k \in \mathcal{K}_j} a_{ijk}^{ts} x_{jk}^t, \qquad \forall i, j, t, s, \\
& x_{jk}^t \in \{0, 1\}, \qquad \forall j, t, k, \\
& z_i^{ts} \in \quad \arg \max_{z_i^{ts} \in \{0,1\}} \quad u_{i0}^{ts}(1 - z_i^{ts}) + U_i^{ts} z_i^{ts} \qquad \forall i, t, s, \\
& U_i^{ts} = \quad \cancel{\max}_{\sigma_i^{ts}} \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts} \qquad \forall i, t, s, \\
& \qquad \text{s.t.} \quad \sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} \leq 1, \qquad \forall i, t, s, r, \\
& \qquad \qquad \sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} \leq 1, \qquad \forall i, t, s, j, \\
& \qquad \qquad \sigma_{ijr}^{ts} \in [0, 1], \qquad \forall i, t, s, j, r.
\end{aligned}
$$

Single-level MIP formulation

$$
\begin{aligned}
\max \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{k'\in\mathcal{K}_j : k'\ge k} x_{jk'}^{t-1} \le \sum_{k'\in\mathcal{K}_j : k'\ge k} x_{jk'}^{t}, \qquad \forall j, t\in\mathcal{T}\setminus\{1\}, k, \\
& \sum_{t'\in\mathcal{T}:t'\le t}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_j} c_{jk}^{t'}(x_{jk}^{t'} - x_{jk}^{t'-1}) \le \sum_{t'\in\mathcal{T}:t'\le t} b^{t'}, \qquad \forall t, \\
& u_{ij}^{ts} = \sum_{k\in\mathcal{K}_j} a_{ijk}^{ts} x_{jk}^t, \qquad \forall i, j, t, s, \\
& x_{jk}^t \in \{0,1\}, \qquad \forall j, t, k, \\
& z_i^{ts} \in \quad \arg\max_{z_i^{ts}\in[0,1]} \quad u_{i0}^{ts}(1 - z_i^{ts}) + U_i^{ts} z_i^{ts} \qquad \forall i, t, s, \\
& U_i^{ts} = \sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{J}} \lambda_{ir} u_{ij}^{ts}\sigma_{ijr}^{ts} \qquad \forall i, t, s, \\
& \sum_{j\in\mathcal{J}} \sigma_{ijr}^{ts} \le 1, \qquad \forall i, t, s, r, \\
& \sum_{r\in\mathcal{J}} \sigma_{ijr}^{ts} \le 1, \qquad \forall i, t, s, j, \\
& \sigma_{ijr}^{ts} \in [0,1], \qquad \forall i, t, s, j, r.
\end{aligned}
$$

Single-level MIP formulation

$$
\begin{aligned}
\max \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{k'\in\mathcal{K}_j : k'\ge k} x_{jk'}^{t-1} \le \sum_{k'\in\mathcal{K}_j : k'\ge k} x_{jk'}^{t}, \qquad \forall j, t\in\mathcal{T}\setminus\{1\}, k, \\
& \sum_{t'\in\mathcal{T}:t'\le t}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_j} c_{jk}^{t'}(x_{jk}^{t'} - x_{jk}^{t'-1}) \le \sum_{t'\in\mathcal{T}:t'\le t} b^{t'}, \qquad \forall t, \\
& u_{ij}^{ts} = \sum_{k\in\mathcal{K}_j} a_{ijk}^{ts} x_{jk}^t, \qquad \forall i, j, t, s, \\
& x_{jk}^t \in \{0,1\}, \qquad \forall j, t, k, \\
& u_{i0}^{ts} z_i^{ts} \le U_i^{ts} z_i^{ts}, \qquad \forall i, t, s, \\
& U_i^{ts} = \sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{J}} \lambda_{ir} u_{ij}^{ts}\sigma_{ijr}^{ts} \qquad \forall i, t, s, \\
& \sum_{j\in\mathcal{J}} \sigma_{ijr}^{ts} \le 1, \qquad \forall i, t, s, r, \\
& \sum_{r\in\mathcal{J}} \sigma_{ijr}^{ts} \le 1, \qquad \forall i, t, s, j, \\
& \sigma_{ijr}^{ts}, z_i^{ts} \in [0,1], \qquad \forall i, t, s, j, r.
\end{aligned}
$$

MILP

$$
\begin{aligned}
\max \quad & \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k \in \mathcal{K}_j} x_{jk}^t \leq 1, \qquad \forall j, t, \\
& \sum_{k' \in \mathcal{K}_j : k' \geq k} x_{jk'}^{t-1} \leq \sum_{k' \in \mathcal{K}_j : k' \geq k} x_{jk'}^{t}, \qquad \forall j, t \in \mathcal{T} \setminus \{1\}, k, \\
& \sum_{t' \in \mathcal{T} : t' \leq t} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} c_{jk}^{t'} (x_{jk}^{t'} - x_{jk}^{t'-1}) \leq \sum_{t' \in \mathcal{T} : t' \leq t} b^{t'}, \qquad \forall t, \\
& u_{ij}^{ts} = \sum_{k \in \mathcal{K}_j} a_{ijk}^{ts} x_{jk}^t, \qquad \forall i, j, t, s, \\
& x_{jk}^t \in \{0, 1\}, \qquad \forall j, t, k, \\
& u_{i0}^{ts} z_i^{ts} \leq U_i^{ts} z_i^{ts}, \quad \forall i, t, s, \\
& U_i^{ts} = \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts}, \qquad \forall i, t, s, \\
& \sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} \leq 1, \quad \forall i, t, s, r, \\
& \sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} \leq 1, \quad \forall i, t, s, j, \\
& \sigma_{ijr}^{ts}, z_i^{ts} \in [0, 1], \quad \forall i, t, s, j, r.
\end{aligned}
$$

[4, 2, 5, 6, 7, 1, 3]

Linearization

$$u_{i0}^{ts} z_i^{ts} \leq \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts} z_i^{ts}, \quad \forall i, t, s,$$

$$\sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} \leq 1, \quad \forall i, t, s, r,$$

$$\sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} \leq 1, \quad \forall i, t, s, j,$$

$$\sigma_{ijr}^{ts}, z_i^{ts} \in [0, 1], \quad \forall i, t, s, j, r.$$

Linearization

1. Apply a perspective transformation. z remains continuous

$$
\begin{aligned}
& u_{i0}^{ts} z_i^{ts} \leq \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts}, \quad \forall i, t, s, \\
& \sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, r, \\
& \sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, j, \\
& \sigma_{ijr}^{ts}, z_i^{ts} \in [0, 1], \quad \forall i, t, s, j, r.
\end{aligned}
$$

[4, 2, 5, 6, 7, 1, 3]

Linearization

1. Apply a perspective transformation. z remains continuous

$$u_{i0}^{ts} z_i^{ts} \leq \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} u_{ij}^{ts} \sigma_{ijr}^{ts}, \quad \forall i, t, s,$$

$$\sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, r,$$

$$\sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, j,$$

$$\sigma_{ijr}^{ts}, z_i^{ts} \in [0, 1], \quad \forall i, t, s, j, r.$$

[4 2 5 6 7 1 3]

Linearization

1. Apply a perspective transformation. z remains continuous

$$
\begin{aligned}
& u_{i0}^{ts} z_i^{ts} \leq \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \lambda_{ir} w_{ijr}^{ts}, \quad \forall i, t, s, \\
& \sum_{j \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, r, \\
& \sum_{r \in \mathcal{J}} \sigma_{ijr}^{ts} \leq z_i^{ts}, \quad \forall i, t, s, j, \\
& z_i^{ts} \in [0, 1], \quad \forall i, t, s, j, r, \\
& \sigma_{ijr}^{ts} \in \{0, 1\}, \quad \forall i, t, s, j, r.
\end{aligned}
$$

2. Standard linearization of bilinear terms with binary variables. σ needs to be integer.

Define new auxiliary variables $w_{ijr}^{ts} := u_{ij}^{ts} \sigma_{ijr}^{ts}, \forall j, r \in \mathcal{J}$, and use constraints:

$$
\begin{aligned}
& w_{ijr}^{ts} \leq M_{ij}^{ts} \sigma_{ijr}^{ts}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}, j, r \in \mathcal{J}, \\
& w_{ijr}^{ts} \leq u_{ij}^{ts}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}, j, r \in \mathcal{J},
\end{aligned}
$$

Valid inequalities and preprocessing

$w_{ijr}^{ts} := u_{ij}^{ts} \sigma_{ijr}^{ts}$

$$w_{ijr}^{ts} := \begin{cases} u_{ij}^{ts} = \sum_{k \in \mathcal{K}_j} a_{ijk}^{ts} x_{jk}^{t}, & u_{ij}^{ts} \text{ is the } r\text{-th largest utility for customer } i, \\ & 0 = a_{ij0}^{ts} \leq a_{ij1}^{ts} \leq \cdots \leq a_{ij|\mathcal{K}_j|}^{ts} \\ 0, & \text{otherwise.} \end{cases}$$

Valid inequalities and preprocessing

$w_{ijr}^{ts} := u_{ij}^{ts} \sigma_{ijr}^{ts}$

$$w_{ijr}^{ts} := \begin{cases} u_{ij}^{ts} = \sum_{k \in \mathcal{K}_j} a_{ijk}^{ts} x_{jk}^{t}, & u_{ij}^{ts} \text{ is the } r\text{-th largest utility for customer } i, \\ & 0 = a_{ij0}^{ts} \leq a_{ij1}^{ts} \leq \cdots \leq a_{ij|\mathcal{K}_j|}^{ts} \\ 0, & \text{otherwise.} \end{cases}$$

$$w_{ijr}^{ts} \leq a_{ijk}^{ts} \sigma_{ijr}^{ts} + \sum_{\substack{k' \in \mathcal{K}_j: \\ k' > k}} (a_{ijk'}^{ts} - a_{ijk}^{ts}) x_{jk'}^{t}, \qquad \forall j, r, k,$$

Valid inequalities and preprocessing

$w_{ijr}^{ts} := u_{ij}^{ts}\sigma_{ijr}^{ts}$

$$w_{ijr}^{ts} := \begin{cases} u_{ij}^{ts} = \sum_{k\in\mathcal{K}_j} a_{ijk}^{ts}x_{jk}^{t}, & u_{ij}^{ts} \text{ is the } r\text{-th largest utility for customer } i, \\ & 0 = a_{ij0}^{ts} \leq a_{ij1}^{ts} \leq \cdots \leq a_{ij|\mathcal{K}_j|}^{ts} \\ 0, & \text{otherwise.} \end{cases}$$

$$w_{ijr}^{ts} \leq a_{ijk}^{ts}\sigma_{ijr}^{ts} + \sum_{\substack{k'\in\mathcal{K}_j: \\ k'>k}} (a_{ijk'}^{ts} - a_{ijk}^{ts})x_{jk'}^{t}, \qquad \forall j, r, k,$$

$$\sum_{r\in\mathcal{J}} w_{ijr}^{ts} \leq \sum_{r\in\mathcal{J}} a_{ijk}^{ts}\sigma_{ijr}^{ts} + \sum_{\substack{k'\in\mathcal{K}_j: \\ k'>k}} (a_{ijk'}^{ts} - a_{ijk}^{ts})x_{jk'}^{t}, \qquad \forall j, k,$$

Valid inequalities and preprocessing

$w_{ijr}^{ts} := u_{ij}^{ts}\sigma_{ijr}^{ts}$

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$$w_{ijr}^{ts} \le a_{ijk}^{ts}\sigma_{ijr}^{ts} + \sum_{\substack{k'\in\mathcal{K}_j:\\ k'>k}} (a_{ijk'}^{ts} - a_{ijk}^{ts})x_{jk'}^{t}, \qquad \forall j, r, k,$$

$$\sum_{r\in\mathcal{J}} w_{ijr}^{ts} \le \sum_{r\in\mathcal{J}} a_{ijk}^{ts}\sigma_{ijr}^{ts} + \sum_{\substack{k'\in\mathcal{K}_j:\\ k'>k}} (a_{ijk'}^{ts} - a_{ijk}^{ts})x_{jk'}^{t}, \qquad \forall j, k,$$

$$\sum_{r\in\mathcal{J}} \sigma_{ijr}^{ts} \le \sum_{k\in\mathcal{K}_j} x_{jk}^{t}, \qquad \forall j,$$

$$\sigma_{ijr}^{ts} = 0 \qquad \forall i, t, s, j, r : \lambda_{ir} = 0.$$

Second approach: Benders' decomposition

Master Problem

$$
\begin{aligned}
\max_{\boldsymbol{x},\boldsymbol{z}} \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{\substack{k'\in\mathcal{K}_j:\\ k'\ge k}} x_{jk'}^{t-1} \le \sum_{\substack{k'\in\mathcal{K}_j:\\ k'\ge k}} x_{jk'}^{t}, \qquad \forall j, t\in\mathcal{T}\setminus\{1\}, k, \\
& \sum_{\substack{t'\in\mathcal{T}:\\ t'\le t}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_j} c_{jk}^{t'}(x_{jk}^{t'} - x_{jk}^{t'-1}) \le \sum_{\substack{t'\in\mathcal{T}:\\ t'\le t}} b^{t'}, \qquad \forall t, \\
& x_{jk}^t \in \{0,1\}, \qquad \forall j, t, k, \\
& u_{ij}^{ts} = \sum_{k\in\mathcal{K}_j} a_{ijk}^{ts} x_{jk}^t, \qquad \forall i, j, t, s, \\
& z_i^{ts} \in \quad \arg\max_{z_i^{ts}} \quad u_{i0}^{ts}(1 - z_i^{ts}) + U_i^{ts} z_i^{ts} \qquad \forall i, t, s, \\
& \qquad\qquad \text{s.t.} \quad z_i^{ts} \in \{0,1\}, \\
& U_i^{ts} = \Phi_{\boldsymbol{\lambda_i}}(u_{i1}^{ts}, \ldots, u_{i|\mathcal{J}|}^{ts}), \qquad \forall i, t, s.
\end{aligned}
$$

Master Problem

$$
\begin{aligned}
\max_{x,z} \quad & \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|} \sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{\substack{k'\in\mathcal{K}_j: \\ k'\ge k}} x_{jk'}^{t-1} \le \sum_{\substack{k'\in\mathcal{K}_j: \\ k'\ge k}} x_{jk'}^t, \qquad \forall j, t\in\mathcal{T}\setminus\{1\}, k, \\
& \sum_{\substack{t'\in\mathcal{T}: \\ t'\le t}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} c_{jk}^{t'} (x_{jk}^{t'} - x_{jk}^{t'-1}) \le \sum_{\substack{t'\in\mathcal{T}: \\ t'\le t}} b^{t'}, \qquad \forall t, \\
& x_{jk}^t \in \{0,1\}, \qquad \forall j, t, k, \\
& u_{ij}^{ts} = \sum_{k\in\mathcal{K}_j} a_{ijk}^{ts} x_{jk}^t, \qquad \forall i, j, t, s, \\
& z_i^{ts} \in \quad \underset{z_i^{ts}}{\arg\max} \quad u_{i0}^{ts}(1 - z_i^{ts}) + U_i^{ts} z_i^{ts} \qquad \forall i, t, s, \\
& \qquad\qquad \text{s.t.} \quad z_i^{ts} \in \{0,1\}, \\
& U_i^{ts} = \Phi_{\boldsymbol{\lambda}_i}(u_{i1}^{ts}, \ldots, u_{i|\mathcal{J}|}^{ts}), \qquad \forall i, t, s.
\end{aligned}
$$

Master Problem

$$
\begin{aligned}
\max_{x,z} \quad & \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|} \sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{\substack{k'\in\mathcal{K}_j:\\ k'\ge k}} x_{jk'}^{t-1} \le \sum_{\substack{k'\in\mathcal{K}_j:\\ k'\ge k}} x_{jk'}^{t}, \qquad \forall j, t\in\mathcal{T}\setminus\{1\}, k, \\
& \sum_{\substack{t'\in\mathcal{T}:\\ t'\le t}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} c_{jk}^{t'} (x_{jk}^{t'} - x_{jk}^{t'-1}) \le \sum_{\substack{t'\in\mathcal{T}:\\ t'\le t}} b^{t'}, \qquad \forall t, \\
& x_{jk}^t \in \{0,1\}, \qquad \forall j, t, k, \\
& u_{0i}^{ts} z_i^{ts} \le {\color{red} U_i^{ts}(x) z_i^{ts}}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, s\in\mathcal{S}.
\end{aligned}
$$

Master Problem

$$
\begin{aligned}
\max_{\boldsymbol{x},\boldsymbol{z}} \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} n_i^t \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}} z_i^{ts} \\
\text{s.t.} \quad & \sum_{k\in\mathcal{K}_j} x_{jk}^t \le 1, \qquad \forall j, t, \\
& \sum_{\substack{k'\in\mathcal{K}_j:\\ k'\ge k}} x_{jk'}^{t-1} \le \sum_{\substack{k'\in\mathcal{K}_j:\\ k'\ge k}} x_{jk'}^{t}, \qquad \forall j, t\in\mathcal{T}\setminus\{1\}, k, \\
& \sum_{\substack{t'\in\mathcal{T}:\\ t'\le t}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_j} c_{jk}^{t'}(x_{jk}^{t'} - x_{jk}^{t'-1}) \le \sum_{\substack{t'\in\mathcal{T}:\\ t'\le t}} b^{t'}, \qquad \forall t, \\
& x_{jk}^t \in \{0,1\}, \qquad \forall j, t, k, \\
& u_{0i}^{ts} z_i^{ts} \le U_i^{ts}(x) z_i^{ts} \le U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_j}\left[\bar{s}_{jk}(x_{jk}^t - \bar{x}_{jk}^t)\right]^+, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, s\in\mathcal{S}.
\end{aligned}
$$

Subproblems

$$u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \left[\bar{s}_{jk}(x_{jk}^t - \bar{x}_{jk}^t)\right]^+$$

$$
\begin{aligned}
\text{(PR)} \quad \max_{\sigma} \quad & \sum_{j\in\mathcal{J}} \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \lambda_r a_{jk} \sigma_{jkr} \\
\text{s.t.} \quad & \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall r, \\
& \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall j, \\
& \sum_{r\in\mathcal{J}} \sigma_{jkr} \leq \bar{x}_{jk}, \quad \forall j, k \\
& \sigma_{jkr} \geq 0, \quad \forall j, r, k.
\end{aligned}
$$

Subproblems

$$u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \left[\bar{s}_{jk}(x_{jk}^t - \bar{x}_{jk}^t) \right]^+$$

$$\begin{aligned}
\text{(PR)} \quad \max_{\sigma} \quad & \sum_{j\in\mathcal{J}} \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \lambda_r a_{jk} \sigma_{jkr} \\
\text{s.t.} \quad & \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall r, \quad \color{red}{(\gamma_r)} \\
& \sum_{r\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall j, \quad \color{red}{(\delta_j)} \\
& \sum_{r\in\mathcal{J}} \sigma_{jkr} \leq \bar{x}_{jk}, \quad \forall j, k \quad \color{red}{(\eta_{jk})} \\
& \sigma_{jkr} \geq 0, \quad \forall j, r, k.
\end{aligned}$$

$$\begin{aligned}
\text{(DU)} \quad \min \quad & \sum_{r\in\mathcal{J}} \gamma_r + \sum_{j\in\mathcal{J}} \delta_j + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}_j} \bar{x}_{jk} \eta_{jk} \\
\text{s.t.} \quad & \gamma_r + \delta_j + \eta_{jk} \geq \lambda_r a_{jk}, \quad \forall j, r \in \mathcal{J}, k \in \mathcal{K}_j.
\end{aligned}$$

Subproblems

$$u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \left[\bar{s}_{jk} (x_{jk}^t - \bar{x}_{jk}^t) \right]^+$$

$$\begin{aligned}
\text{(PR)} \quad \max_{\sigma} \quad & \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \lambda_r a_{jk} \sigma_{jkr} \\
\text{s.t.} \quad & \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall r, \\
& \sum_{r \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall j, \\
& \sum_{r \in \mathcal{J}} \sigma_{jkr} \leq \bar{x}_{jk}, \quad \forall j, k \\
& \sigma_{jkr} \geq 0, \quad \forall j, r, k.
\end{aligned}$$

$$\begin{aligned}
\text{(DU)} \quad \min \quad & \sum_{r \in \mathcal{J}} \gamma_r + \sum_{j \in \mathcal{J}} \delta_j + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \bar{x}_{jk} \eta_{jk} \\
\text{s.t.} \quad & \gamma_r + \delta_j + \eta_{jk} \geq \lambda_r a_{jk}, \quad \forall j, r \in \mathcal{J}, k \in \mathcal{K}_j.
\end{aligned}$$

Subproblems

$$u_{0i}^{ts} z_i^{ts} \leq U_i^{ts}(\bar{x}) z_i^{ts} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \left[\bar{s}_{jk} (x_{jk}^t - \bar{x}_{jk}^t) \right]^+$$

$$\begin{aligned}
\text{(PR)} \quad \max_{\sigma} \quad & \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \lambda_r a_{jk} \sigma_{jkr} \\
\text{s.t.} \quad & \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall r, \\
& \sum_{r \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \sigma_{jkr} \leq 1, \quad \forall j, \\
& \sum_{r \in \mathcal{J}} \sigma_{jkr} \leq \bar{x}_{jk}, \quad \forall j, k \\
& \sigma_{jkr} \geq 0, \quad \forall j, r, k.
\end{aligned}$$

$$\begin{aligned}
\text{(DU)} \quad \min \quad & \sum_{r \in \mathcal{J}} \gamma_r + \sum_{j \in \mathcal{J}} \delta_j + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_j} \bar{x}_{jk} \eta_{jk} \\
\text{s.t.} \quad & \gamma_r + \delta_j + \eta_{jk} \geq \lambda_r a_{jk}, \quad \forall j, r \in \mathcal{J}, k \in \mathcal{K}_j.
\end{aligned}$$

Tailored Primal-Dual algorithm to solve (PR)-(DU) at each iteration.

Computational Experiments

1. Computational Performance using synthetic data

Instance sizes and other parameters

Facilities and types: $|\mathcal{L}| \in \{10, 30\}$, $|\mathcal{K}_j| = 4$.
Customers: $|\mathcal{I}| \in \{20, 30, 40, 50\}$.
Scenarios: $|\mathcal{S}| \in \{5, 10\}$.
Budget and time period: $|\mathcal{T}| = 3$, $b^t \in \{5, 10\}$.
$u_0 \in \{10, 12, 15\}$.
Five instances of each size, a total of 1680.
Time limit equal to 3600 seconds.
The total number of instances per row is 30.

Type `C`**:** $\boldsymbol{\lambda} = (1, 0, \ldots, 0)$. Standard RUM.

Type `G`**:** $\boldsymbol{\lambda} = (1, \frac{1}{9}, \frac{1}{27}, 0, \ldots, 0)$. Geometric rule.

Type `K`**:** $\boldsymbol{\lambda} = (1, 1, 0, \ldots, 0)$. Aggregation with *consideration set* of size 2.

Type `L`**:** $\boldsymbol{\lambda} = (1, \frac{1}{2}, 0, \ldots, 0)$. Linear rule.

Comparison of formulations (10 scenarios)

SL: MILP SL+VI: MILP with valid ineq. B: Benders Decomposition

$\|\mathcal{I}\|$	$\boldsymbol{\lambda}$	$\|\mathcal{J}\|$	Time[s] (#Solved)			MIPGap[%]		
			SL	SL+VI	B	SL	SL+VI	B
30	C	10	0.9 (30)	11.7 (30)	6.3 (30)	0.0	0.0	0.0
		30	6.7 (30)	112.0 (25)	49.2 (25)	0.0	27.2	2.0
	G	10	1890.4 (1)	1210.5 (13)	671.5 (20)	56.3	27.3	14.8
		30	TL (0)	TL (0)	TL (0)	126.7	219.0	47.0
	K	10	2539.9 (9)	355.5 (30)	174.6 (28)	14.9	0.0	14.5
		30	TL (0)	3365.0 (2)	475.2 (14)	38.9	14.4	10.3
	L	10	1548.3 (11)	791.1 (18)	665.3 (28)	23.4	21.5	6.7
		30	TL (0)	TL (0)	1151.3 (5)	86.1	50.7	31.8
50	C	10	1.6 (30)	30.1 (30)	14.7 (30)	0.0	0.0	0.0
		30	13.6 (30)	215.2 (20)	73.0 (18)	0.0	44.3	8.6
	G	10	TL (0)	1597.1 (5)	338.6 (10)	56.6	33.2	20.3
		30	TL (0)	TL (0)	TL (0)	166.8	404.1	67.1
	K	10	3128.0 (1)	656.3 (22)	299.3 (21)	18.5	12.7	6.8
		30	TL (0)	TL (0)	1184.4 (12)	59.7	23.8	17.9
	L	10	2723.4 (3)	1421.0 (13)	1191.6 (17)	31.3	22.6	7.7
		30	TL (0)	TL (0)	2659.9 (4)	101.5	61.8	33.6

Comparison of formulations (10 scenarios)

SL: MILP SL+VI: MILP with valid ineq. B: Benders Decomposition

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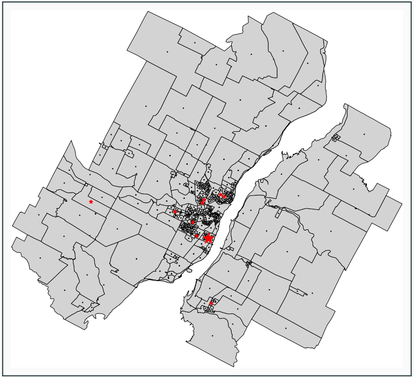
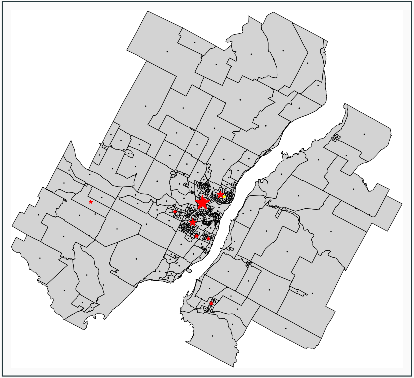
Case Study: Electric Vehicle Charging Stations in Trois-Rivières, Canada

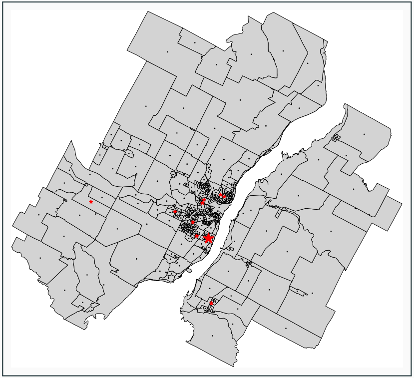


(a) $\boldsymbol{\lambda} = C = (1, 0, \ldots, 0)$

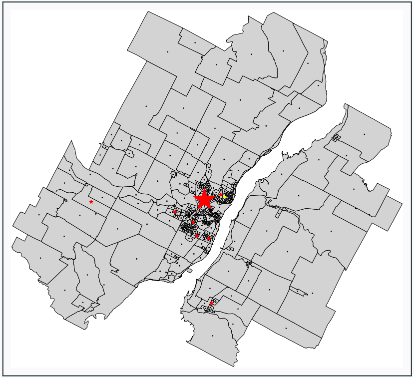


(b) $\boldsymbol{\lambda} = K = (1, 1, 0, \ldots, 0)$

Case Study: Electric Vehicle Charging Stations in Trois-Rivières, Canada



(a) $\boldsymbol{\lambda} = C = (1, 0, \ldots, 0)$



(c) $\boldsymbol{\lambda} = L = (1, \frac{1}{2}, 0, \ldots, 0)$

Conclusions and future work

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The CMCFL introduces a **cooperative framework** for the MCFL with binary choice rule.

Conclusions and future work

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The CMCFL introduces a **cooperative framework** for the MCFL with binary choice rule.

Future work

- Add **capacities** to the facilities dependent on the type of plant installed $\Rightarrow$ Bilevel location-allocation problem.
- Robustify the OMf associated to the utility. For instance considering **non-monotone or negative λ-weights**, or **variable λ-weights** that meet certain conditions associated to the knowledge of the customer, and optimize U in the worst-case.

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Thank you for your attention!

Questions?

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