On warm-starting constraint generation methods via machine learning tools for solving mixed-integer programs

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Outline

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Mixed Integer
Linear Programs
(MILP)

Machine
Learning
(ML)

Combine knowledge from both worlds

Recent reviews: *Bengio et al. [2021]; Gambella et al. [2021]*

Outline

MILP

$$(P_{\boldsymbol{\theta}}[\mathcal{J}]) \qquad \begin{cases} \min\limits_{\boldsymbol{z} \in \mathbb{R}^n \times \mathbb{Z}^q} \boldsymbol{c}^T \boldsymbol{z} \\ \quad \text{s.t. } \boldsymbol{a}_j^T \boldsymbol{z} \leq b_j, \quad \forall j \in \mathcal{J} \end{cases}$$

- $\boldsymbol{\theta} = \{\boldsymbol{c}, \boldsymbol{a}_j, b_j, \forall j \in \mathcal{J}\}$.
- $P_{\boldsymbol{\theta}}[\mathcal{J}]$ bounded and feasible.
- Optimal solution $\boldsymbol{z}^*_{\boldsymbol{\theta}}[\mathcal{J}]$ is a singleton.

MILP

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- NP-hard. Very difficult to solve.
- Pure optimization-based strategy: Constraint generation (CG), *Minoux [1989]*.
- Iterative process. Unaffordable in online applications.

MILP

$$(P_{\boldsymbol{\theta}}[\mathcal{J}]) \qquad \begin{cases} \min\limits_{\boldsymbol{z} \in \mathbb{R}^n \times \mathbb{Z}^q} \boldsymbol{c}^T \boldsymbol{z} \\ \quad \text{s.t. } \boldsymbol{a}_j^T \boldsymbol{z} \leq b_j, \quad \forall j \in \mathcal{J} \end{cases}$$

- Apply ML to alleviate computational burden in CG.
- Enhacing performance in MILPs.
- Learning from the information of previously solved instances.
- Providing a *good* warm-start.
- Reducing number of iterations in CG.
- Reducing computational time in MILPs.

MILP

$$(P_{\boldsymbol{\theta}}[\mathcal{J}]) \qquad \begin{cases} \min\limits_{\boldsymbol{z} \in \mathbb{R}^n \times \mathbb{Z}^q} \boldsymbol{c}^T \boldsymbol{z} \\ \quad \text{s.t.} \ \boldsymbol{a}_j^T \boldsymbol{z} \leq b_j, \quad \forall j \in \mathcal{J} \end{cases}$$

- Literature: only use information from the binding constraints, i.e., $\boldsymbol{a}_j^T \boldsymbol{z}^* = b_j$.
- Not enough when integer variables appear.
- Some non-binding constraints should be included. Need to define *invariant constraint set*.

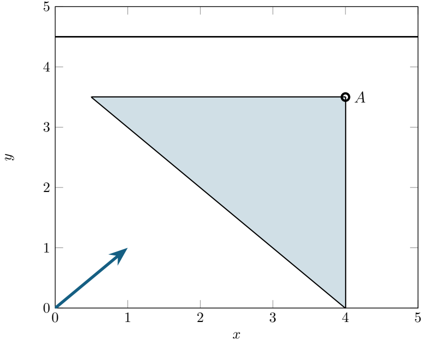
Invariant Constraint Set, $\mathcal{S}$

According to *Calafiore [2010]*:

$\mathcal{S} \subset \mathcal{J}$ s.t. $\boldsymbol{c}^T \boldsymbol{z}^*_{\boldsymbol{\theta}}[\mathcal{S}] = \boldsymbol{c}^T \boldsymbol{z}^*_{\boldsymbol{\theta}}[\mathcal{J}]$

The integrality of the decision variables is crucial to find out which constraints belong to $\mathcal{S}$.

LP vs MILP (Example taken from *Pineda et al. [2020]*)



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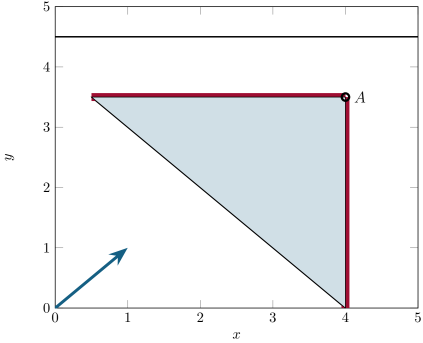

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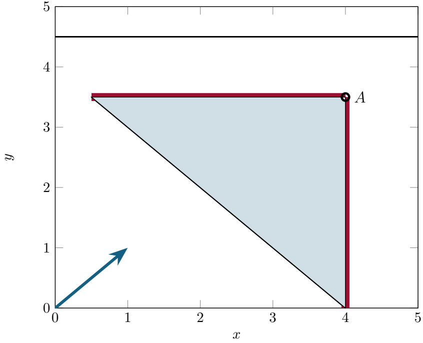

Binding constraints

$$\mathcal{B} = \{ j \in \mathcal{J} : \boldsymbol{a}_j^T \boldsymbol{z}_\theta^*[\mathcal{J}] = b_j \}$$

$$\mathcal{S} = \mathcal{B}$$

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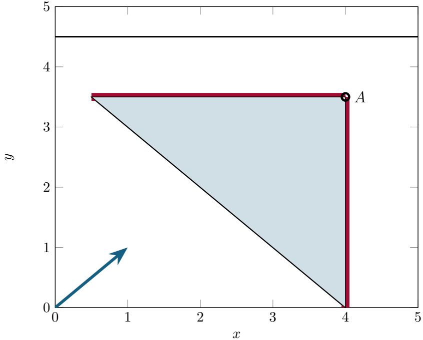

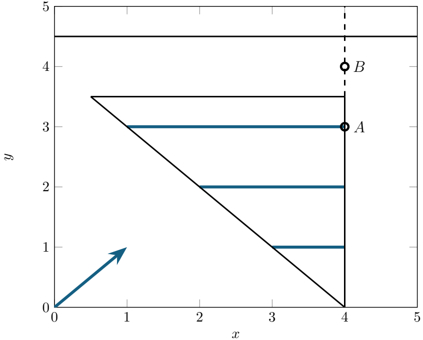

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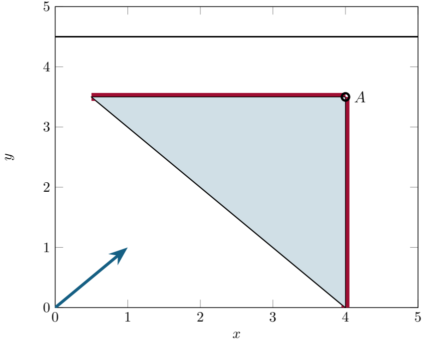

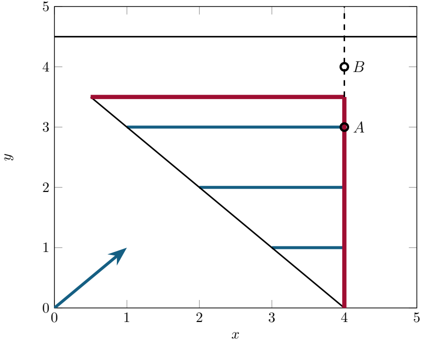

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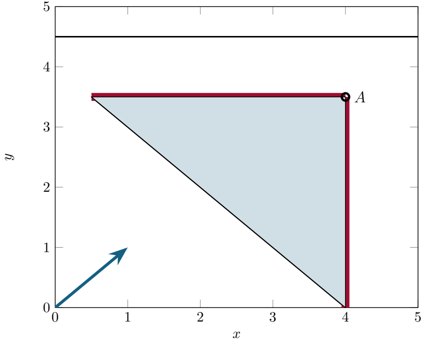
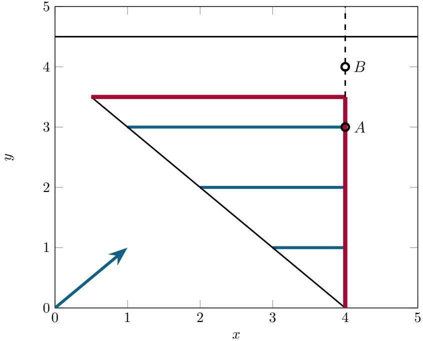




Binding constraints

$$\mathcal{B} = \{j \in \mathcal{J} : \boldsymbol{a}_j^T \boldsymbol{z}_\theta^*[\mathcal{J}] = b_j\}$$

Some non-binding constraints

also belong to $\mathcal{S}$.

$$\mathcal{S} = \mathcal{B}$$

$$\mathcal{S} \supset \mathcal{B}$$

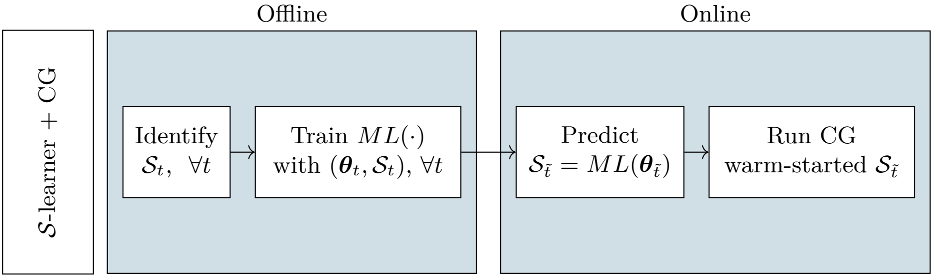
- Finding $\mathcal{S}$ is challenging in MILPs.
- For each train instance t, we look for $\mathcal{S}_t$, including some of the non-binding constraints.
- And reduced MILP $P_{\theta_t}[\mathcal{S}_t]$ is solved.

How to find $\mathcal{S}_t$?

Algorithm Identifying an invariant constraint set for each instance t

0) Initialize $\mathcal{S}_t = \mathcal{B}_t$.
1) Solve $P_{\boldsymbol{\theta_t}}[\mathcal{S}_t]$ with solution $\boldsymbol{z}^*_{\boldsymbol{\theta}_t}[\mathcal{S}_t]$.
2) If $\boldsymbol{z}^*_{\boldsymbol{\theta}_t}[\mathcal{S}_t]$ is infeasible for $P_{\boldsymbol{\theta_t}}[\mathcal{J}]$, go to step 3). Otherwise, stop.
3) $\mathcal{S}_t := \mathcal{S}_t \cup \{j \in \mathcal{J} \setminus \mathcal{S}_t : j \text{ is the most violated constraint}\}$, go to step 1).

Recap


$\mathcal{S}$-learner + CG
Offline
Identify $\mathcal{S}_t, \ \forall t$
Train $ML(\cdot)$ with $(\boldsymbol{\theta}_t, \mathcal{S}_t), \forall t$
Online
Predict $\mathcal{S}_{\tilde{t}} = ML(\boldsymbol{\theta}_{\tilde{t}})$
Run CG warm-started $\mathcal{S}_{\tilde{t}}$

Advantages

- Based on CG. Crucial constraints are included.
- Optimality and feasibility guarantees of CG are preserved.
- Reducing number of iterations of CG.
- Maybe only one iteration.
- Independent on the ML method used.
- Identifying $\mathcal{S}$ and training ML is performed offline.

Outline

Experimental Setup

- Binary classification problem. $k$nn.
- Label $s_j^t = \pm 1$ depending on inclusion on $\mathcal{S}_t$.
- We choose to be conservative when including constraints.
- Synthetic and real-world applications.

Two comparative approaches

	Offline	Online
CG		Run CG no warm-start
$\mathcal{B}$-learner + CG	Identify $\mathcal{B}_t, \ \forall t$ → Train $ML(\cdot)$ with $(\boldsymbol{\theta}_t, \mathcal{B}_t), \forall t$ →	Predict $\mathcal{B}_{\tilde{t}} = ML(\boldsymbol{\theta}_{\tilde{t}})$ → Run CG warm-started $\mathcal{B}_{\tilde{t}}$
$\mathcal{S}$-learner + CG	Identify $\mathcal{S}_t, \ \forall t$ → Train $ML(\cdot)$ with $(\boldsymbol{\theta}_t, \mathcal{S}_t), \forall t$ →	Predict $\mathcal{S}_{\tilde{t}} = ML(\boldsymbol{\theta}_{\tilde{t}})$ → Run CG warm-started $\mathcal{S}_{\tilde{t}}$

Unit Commitment problem

$$\begin{cases} \min\limits_{\boldsymbol{x}\in\mathbb{R}^n,\, \boldsymbol{y}\in\{0,1\}^n} \sum_{i=1}^{n} c_i x_i \\ \quad \text{s.t.} \ \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} d_i, \\ \quad -f_j \leq \sum_{i=1}^{n} a_{ij}(x_i - d_i) \leq f_j, \quad j = 1, \ldots, m \\ \quad l_i y_i \leq x_i \leq u_i y_i, \quad i = 1, \ldots, n \end{cases}$$

- $\boldsymbol{\theta} = \mathbf{d}$.
- $n = 96$.
- $m = 120$ (240 constraints).
- $T = 8640$ (Leave-one-out).

Results

	k	$[C_{min}, C_{max}]$	$[I_{min}, I_{max}]$	$P_1(\%)$	$\Delta(\%)$
CG	-	[0, 22]	[1, 23]	9.16	188.38
$\mathcal{B}$+CG	5	[0, 23]	[1, 8]	54.40	74.09
	20	[0, 26]	[1, 5]	68.28	62.35
	100	[0, 29]	[1, 5]	83.70	54.62
$\mathcal{S}$+CG	5	[0, 26]	[1, 5]	92.66	44.84
	20	[0, 29]	[1, 4]	98.81	42.83
	100	[0, 32]	[1, 3]	99.71	44.41

• C : constr. • I: iterat. • P_1: one iteration. • Δ: time wrt MILP solver.

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Pure optimization-based tools worse than MILP solver

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Larger values of k imply more constraints (more conservative).

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Computational gains in both approaches.

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Few extra constraints in S-learner. Large time improvements.

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Adding constraints is not enough ($k = 20$ vs $k = 5$)

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• C : constr. • I: iterat. • P_1: one iteration. • Δ: time wrt MILP solver.

$\approx$ 100% of the instances are optimally solved with one iteration (P_1).

More details

Machine-learning-aided warm-start of constraint generation methods for online mixed-integer optimization

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Abstract

Mixed Integer Linear Programs (MILP) are well known to be NP-hard problems in general. Even though pure optimization-based methods, such as constraint generation, are guaranteed to provide an optimal solution if enough time is given, their use in online applications is still a great challenge due to their usual excessive time requirements. To alleviate their computational burden, some machine learning techniques have been proposed in the literature, using the information provided by previously solved MILP instances. Unfortunately, these techniques report a non-negligible percentage of infeasible or suboptimal instances.

By linking mathematical optimization and machine learning, this paper proposes a novel approach that speeds up the traditional constraint generation method, preserving feasibility and optimality guarantees. In particular, we first identify offline the so-called invariant constraint set of past MILP instances. We then train (also offline) a machine learning method to learn an invariant constraint set as a function of the problem parameters of each instance. Next, we predict online an invariant constraint set of the new unseen MILP application and use it to initialize the constraint generation method. This warm-started strategy significantly reduces the number of iterations to reach optimality, and therefore, the computational burden to solve online each MILP problem is significantly reduced. Very importantly, the proposed methodology inherits the feasibility and optimality guarantees of the traditional constraint generation method. The computational performance of the proposed approach is quantified through synthetic and real-life MILP applications.

Keywords: Mixed integer linear programming, machine learning, constraint generation, warm-start, feasibility and optimality guarantees

Available at:

https://www.researchgate.net/publication/350371853_Machine-learning-aided_warm-start_of_constraint_generation_methods_for_online_mixed-integer_optimization

SCAN ME

Outline

Conclusions

- Approach which combines MILPs and ML.
- Warm-start in CG algorithm.
- Keeping optimality and feasibility guarantees of CG.
- Reduce computational burden.
- Tested on synthetic and real-world applications.

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- Warm-start in CG algorithm.
- Keeping optimality and feasibility guarantees of CG.
- Reduce computational burden.
- Tested on synthetic and real-world applications.

Further research

- Other input parameters.
- Introduce expert-knowledge information.

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Thank you very much for your attention!

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