Chance-Constrained Optimization applied to the Optimal Power Flow problem

A MIP approach

Álvaro Porras, *Concepción Domínguez*, Juan Miguel Morales, Salvador Pineda

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OASYS Group, Universidad de Málaga

Outline

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General chance-constrained SAA MIP reformulation

Tightening and Screening

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Introduction to Chance-Constrained Problems

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- **Optimal Power Flow**: minimize the expected operating cost whilst guaranteeing that the system withstands unforeseen peeks of electrical load due to stochastic demand.
- General (linear) formulation:

$$\begin{aligned} \min_{x} \quad & c^\top x \\ \text{s.t.} \quad & x \in X \\ & \mathbb{P}\left\{ a_j(\omega)^\top x \leqslant b_j(\omega),\ \forall j \right\} \geqslant 1 - \epsilon. \end{aligned}$$

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Tightening and Screening

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Algorithm 1 Iterative Coefficient Strengthening (κ iterations)

Initialization: $k \leftarrow 0$, $M_{js}^{0} \leftarrow \infty$.
while $k < \kappa$ **do**
  for $j \in \mathcal{J}$ and $s \in \mathcal{S}$ **do**
    if $M_{js}^{k} > 0$ **then**
      1) Tightening phase: Solve

$$\begin{aligned} M_{js}^{k+1} \leftarrow \arg\max_{x,y} \quad & a_{js}^{\top} x - b_{js} \\ \text{s.t.} \quad & x \in X \\ & x^{\top} a_j^0 + \Omega_s \hat{a}_j^{\top} x - b_{js} \leqslant M_{js}^{k} y_s, \quad \forall j, s \\ & \sum_{s \in \mathcal{S}} y_s \leqslant p \\ & 0 \leqslant y_s \leqslant 1, \quad \forall s. \end{aligned}$$

    end if
    if $M_{js}^{k+1} < 0$ **then**
      2) Screening phase: Eliminate constraint (j, s) from the model.
    end if
  end for
  Set $k \leftarrow k + 1$.
end while

Valid inequalities

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For a fixed j:

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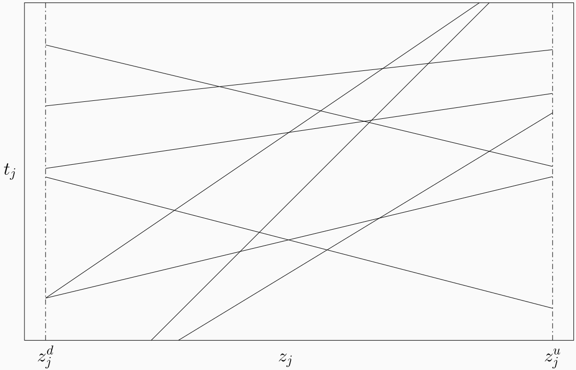
$z_j := \hat{a}_j^\top x$, $z_j \in [z_j^d, z_j^u]$:

$$L_{js} : f_{js}(z_j) = \Omega_s z_j - b_{js}$$

Valid inequalities



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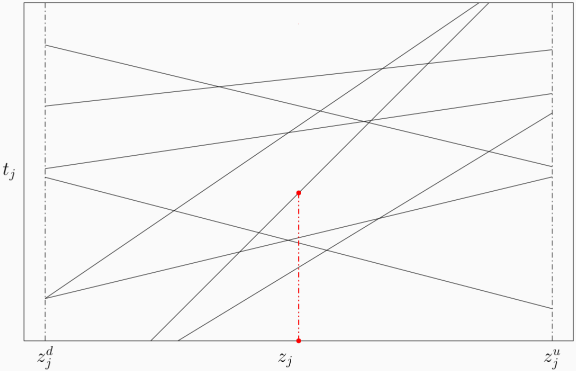

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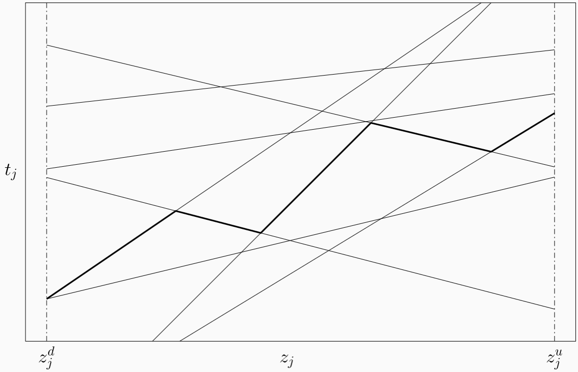


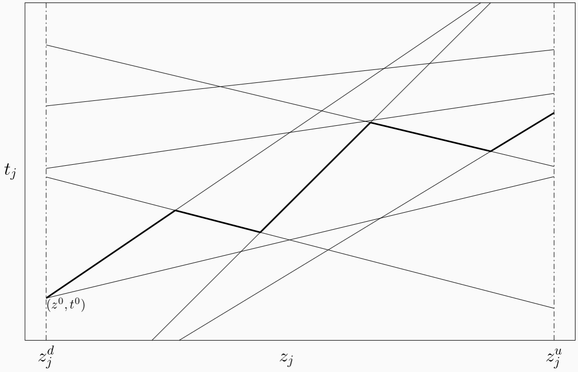
Proposition

For a fixed $j \in \mathcal{J}$, let $U_j^{p+1}(\cdot)$ be the (p+1)-upper envelope of the set of lines $\mathcal{L}_j$, with $p := \lfloor \epsilon |\mathcal{S}| \rfloor$. Then the inequality

$$U_j^{p+1}(\hat{a}_j^\top x) + x^\top a_j^0 \leqslant 0, \quad x \in X$$

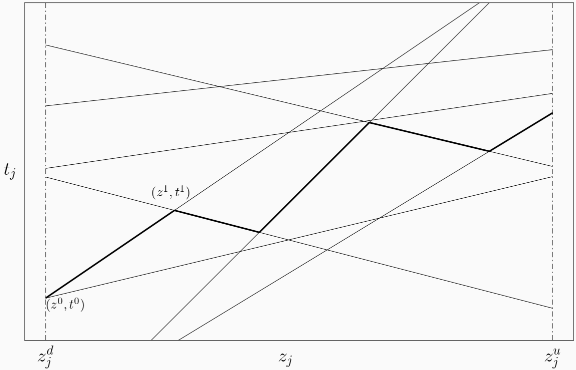
is valid.

Valid inequalities



Rider Algorithm

Valid inequalities



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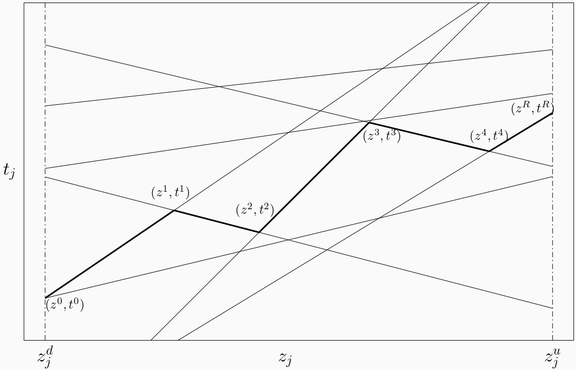

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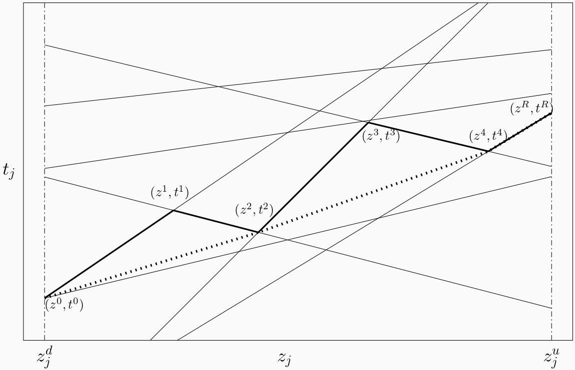
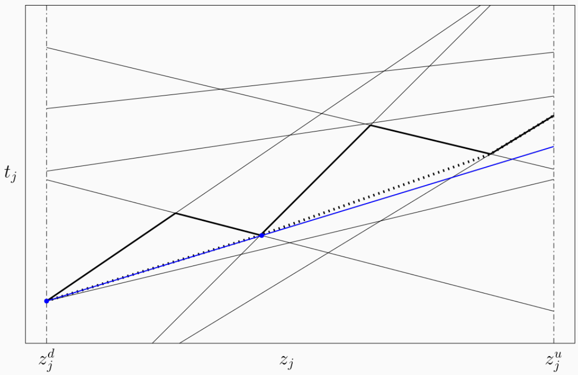


Lower hull: *Jarvis March*, *Graham scan*

Valid inequalities



Corollary

Let $\{(z^r, t^r)\}$, $r \in \{0, \ldots, R\}$, be an ordered set of vertices, and let $\{(z^{r'}, t^{r'})\}$, $r' \in \{0, \ldots, R'\} \subseteq \{0, \ldots, R\}$, be the ordered subset of vertices such that the associated polygonal chain is the lower hull. Then the following linear inequalities are valid:

$$\frac{t^{r'+1} - t^{r'}}{z^{r'+1} - z^{r'}}(\hat{a}_j^\top x - z^{r'}) + t^{r'} \leqslant -x^\top a_j^0, \quad x \in X, r' \in \{0, \ldots, R' - 1\}$$

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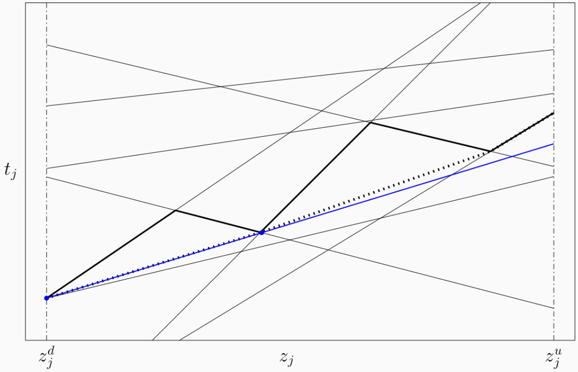

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- Relationship with *quantile cuts*.

Computational Results: OPF

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- Approaches **T**, **TS**, **V** and **TS+V** using five standard power systems.
- Instance: IEEE-300 test system: 300 nodes, 57 generators, 411 lines.
- GUROBI 9.1.2 on a Linux-based server with CPUs clocking at 2.6 GHz, 6 threads and 32 GB of RAM.
- 1000 scenarios, 5% violation of the JCC ($\epsilon = 0.05$, $p = 50$).
- Time limit: 10 hours.
- Results averaged over ten instances.

Computational Results

IEEE-300	**BN**	**T**(3)	**TS**(3)	**BN+V**	**TS**(1)+**V**
#CON	936939	100%	8.0%	101.5%	3.30%
LRgap	1.114%	0.264%	0.264%	0.3192%	0.1603%
MIPgap	0.27% (0)	0.07% (0)	0.01% (4)	0.08% (0)	0.00% (10)
Time	36000	1.0x	1.2x	1.0x	8.5x

References

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