Tight and Compact Sample Average Approximation for Joint Chance Constrained Optimal Power Flow

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Outline

Introduction to Joint Chance Constrained Optimal Power Flow (JCC-OPF)

- Sample Average Approximation MIP reformulation

Tightening and screening

Valid inequalities

Computational Results: OPF

Introduction to Joint Chance Constrained Optimal Power Flow (JCC-OPF)

Introduction to JCC-OPF

- **OPF under uncertainty**: minimize the expected operating cost whilst guaranteeing that the system withstands unforeseen peeks of electrical load due to stochastic demand.
- Chance constraints ensure feasibility of the system with a **tolerable probability of constraint violation**.
- Cost-effective decisions can be taken by discarding extreme events or unexpected random circumstances.
- General (linear) formulation:

$$\begin{aligned} \min_{x} \quad & c^\top x \\ \text{s.t.} \quad & x \in X \\ & \mathbb{P}\left\{ a_j(\omega)^\top x \leqslant b_j(\omega),\ \forall j \right\} \geqslant 1 - \epsilon. \end{aligned}$$

Introduction to JCC-OPF

Assumptions considered:

- **Net demand uncertainty**: $\tilde{d}_n = d_n - \omega_n$, where d_n where d_n is the predicted value and ω_n is the forecast error with a change of sign.
- **Generation**: To cope with the forecast errors $(\omega_n)_{n \in \mathcal{N}}$, generators' power outputs are adjusted according to the following affine control policy:

$$\tilde{p}_g = p_g - \beta_g \Omega, \quad \forall g \in \mathcal{G},$$

  where $\Omega := \sum_{n \in \mathcal{N}} \omega_n$ is the system-wise aggregated forecast error, and p_g and β_g are the power output dispatch and the participation factor of generating unit g.
- **Power Balance**:

$$\sum_{g \in \mathcal{G}} \tilde{p}_g - \sum_{n \in \mathcal{N}} \tilde{d}_n = \sum_{g \in \mathcal{G}} (p_g - \beta_g \Omega) - \sum_{n \in \mathcal{N}} (d_n - \omega_n) = 0.$$

$$\sum_{g \in \mathcal{G}} p_g - \sum_{n \in \mathcal{N}} d_n = 0 \text{ and } \sum_{g \in \mathcal{G}} \beta_g = 1.$$

- **DC Power Flow**.

Introduction to JCC-OPF

The mathematical formulation is expressed as follows:

$$
\begin{aligned}
\min_{p_g,\beta_g} \quad & \mathbb{E}\left[\sum_{g\in\mathcal{G}} C(p_g,\beta_g)\right] \\
\text{s.t.} \ & \sum_{g\in\mathcal{G}} \beta_g = 1 \\
& \sum_{g\in\mathcal{G}} p_g - \sum_{n\in\mathcal{N}} d_n = 0 \\
& \mathbb{P}\left(\begin{array}{l} \underline{p}_g \leqslant p_g - \Omega\beta_g \leqslant \overline{p}_g, \quad \forall g\in\mathcal{G} \\ -\overline{f}_l \leqslant \sum_{n\in\mathcal{N}} B_{ln}\left(\sum_{g\in\mathcal{G}_n}(p_g - \Omega\beta_g) + \omega_n - d_n\right) \leqslant \overline{f}_l, \quad \forall l\in\mathcal{L} \end{array}\right) \geqslant 1-\epsilon, \\
& p_g,\beta_g \geqslant 0, \forall g\in\mathcal{G}.
\end{aligned}
$$

Chance-constrained SAA MIP reformulation (with Big-Ms)

- Sample Average Approx.: assume a finite discrete distribution $\Rightarrow$ MIP reformulation
- $s \in \mathcal{S}$ scenarios (with equal probabilities)
- $y_s \in \{0,1\}$ such that $y_s = 0 \Leftrightarrow$ the scenario s is satisfied

$$
\begin{aligned}
\min_x \quad & c^\top x \\
\text{s.t.} \quad & x \in X \\
& \mathbb{P}\left\{a_j(\omega)^\top x \leqslant b_j(\omega),\ \forall j\right\} \geqslant 1 - \epsilon. \\
& \sum_{s \in \mathcal{S}} y_s \leqslant p \\
& y_s \in \{0,1\}, \quad \forall s.
\end{aligned}
$$

$$
\begin{aligned}
\min_x \quad & c^\top x \\
\text{s.t.} \quad & x \in X \\
& a_{js}^\top x \leqslant b_j(\omega) + M_{js} y_s, \quad \forall j, s \\
& \sum_{s \in \mathcal{S}} y_s \leqslant p \\
& y_s \in \{0,1\}, \quad \forall s.
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$$

JCC-OPF via Sample Average Approximation

The MIP reformulation of JCC-OPF writes as follows:

$$
\begin{aligned}
\min_{p_g, \beta_g} \quad & \mathbb{E}\left[\sum_{g \in \mathcal{G}} C(p_g, \beta_g)\right] \\
\text{s.t.} \; & \sum_{g \in \mathcal{G}} \beta_g = 1 \\
& \sum_{g \in \mathcal{G}} p_g - \sum_{n \in \mathcal{N}} d_n = 0 \\
& - y_s M^1_{gs} + \underline{p}_g \leqslant p_g - \Omega_s \beta_g \leqslant \overline{p}_g + y_s M^2_{gs}, \quad \forall g, s \\
& - y_s M^3_{ls} - \overline{f}_l \leqslant \sum_{n \in \mathcal{N}} B_{ln} \left(\sum_{g \in \mathcal{G}_n} (p_g - \Omega_s \beta_g) - d_n + \omega_{ns} \right) \leqslant \overline{f}_l + y_s M^4_{ls}, \quad \forall l, s \\
& p_g, \beta_g \geqslant 0, \forall g \in \mathcal{G}. \\
& \sum_{s \in \mathcal{S}} y_s \leqslant p \\
& y_s \in \{0, 1\}, \quad \forall s.
\end{aligned}
$$

Tightening and screening

Tightening and screening

Algorithm 1 Iterative Coefficient Strengthening (κ iterations)

Initialization: $k \leftarrow 0$, $M_{js}^0 \leftarrow \infty$.
while $k < \kappa$ **do**
  for $j \in \mathcal{J}$ and $s \in \mathcal{S}$ **do**
    if $M_{js}^k > 0$ **then**
      1) Tightening phase: Solve

$$\begin{aligned} M_{js}^{k+1} \leftarrow \arg\max_{x,y} \quad & a_{js}^\top x - b_{js} \\ \text{s.t.} \quad & x \in X \\ & a_{js}^\top x - b_{js} \leqslant M_{js}^k y_s, \quad \forall j, s \\ & \sum_{s \in \mathcal{S}} y_s \leqslant p \\ & 0 \leqslant y_s \leqslant 1, \quad \forall s. \end{aligned}$$

    end if
    if $M_{js}^{k+1} < 0$ **then**
      2) Screening phase: Eliminate constraint (j, s) from the model.
    end if
  end for
  Set $k \leftarrow k + 1$.
end while

Valid inequalities

Valid inequalities I: generators

For a given g (the process is analogous for the upper bound constraint)

- $p_g - \Omega_s \beta_g \geqslant \underline{p}_g \quad \Rightarrow \quad p_g - \underline{p}_g \geqslant \Omega_s \beta_g$
- Let us consider a set of scenarios $\Omega := \{2, -4, 4, 5, 0\}$
- Acceptable violation probability is 40%. Then, if we have 5 scenarios, a constraint can be violated at most in 2 scenarios.

$$p_g - \underline{p}_g \geqslant 2\,\beta_g$$

$$p_g - \underline{p}_g \geqslant -4\,\beta_g$$

$$p_g - \underline{p}_g \geqslant 4\,\beta_g$$

$$p_g - \underline{p}_g \geqslant 5\,\beta_g$$

$$p_g - \underline{p}_g \geqslant 0\,\beta_g$$

*It is like ordering 1-dimensional affine functions without intercept.

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- In a descending order, the constraint with the third largest Ω must be satisfied.
- Therefore, the following constraint is a valid inequality.

$$p_g - \underline{p}_g \geqslant 2\,\beta_g$$

*It is like ordering 1-dimensional affine functions without intercept.

Valid inequalities II: lines

For a given l (the process is analogous for the upper bound constraint)

$$\sum_{n \in \mathcal{N}} B_{ln} \left(\sum_{g \in \mathcal{G}_n} (p_g - \Omega_s \beta_g) + \omega_{ns} - d_n \right) \geqslant -\overline{f}_l$$

$$\sum_{n \in \mathcal{N}} B_{ln} \left(\sum_{g \in \mathcal{G}_n} p_g - d_n \right) + \overline{f}_l \geqslant \Omega_s \sum_{n \in \mathcal{N}} B_{ln} \sum_{g \in \mathcal{G}_n} \beta_g - \sum_{n \in \mathcal{N}} B_{ln} \omega_{ns}$$

Per each line constraint, the right-hand side is an 1-dimensional affine function per scenario as follows:

$$t_{ls} = \Omega_s z_l + b_{ls}$$

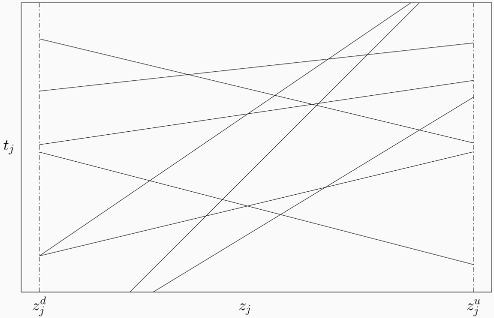
where:

$$z_l = \sum_{n \in \mathcal{N}} B_{ln} \sum_{g \in \mathcal{G}_n} \beta_g \quad \text{and} \quad \underline{z}_l \leqslant z_l \leqslant \overline{z}_l$$

$$b_{ls} = \sum_{n \in \mathcal{N}} B_{ln} \omega_{ns}$$

Valid inequalities II: lines

$$\sum_{n\in\mathcal{N}} B_{ln}\left(\sum_{g\in\mathcal{G}_n} p_g - d_n\right) + \overline{f}_l \geqslant t_{ls} = \Omega_s z_l + b_{ls}$$



Let us consider an example of 8 scenarios where the constraints can be violated in at most 4 scenarios.

Valid inequalities II: lines

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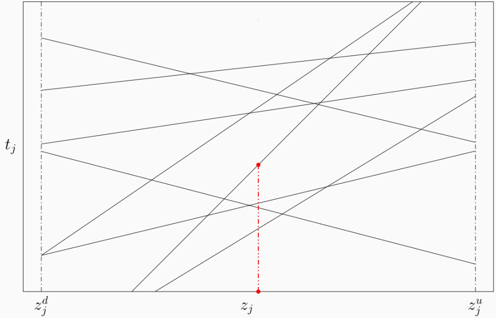

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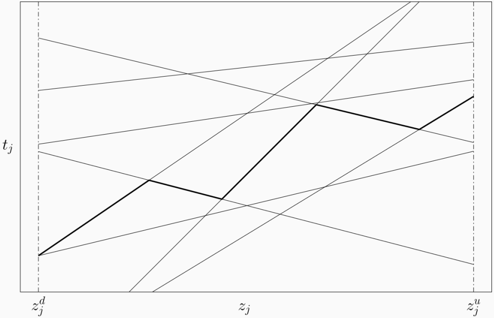

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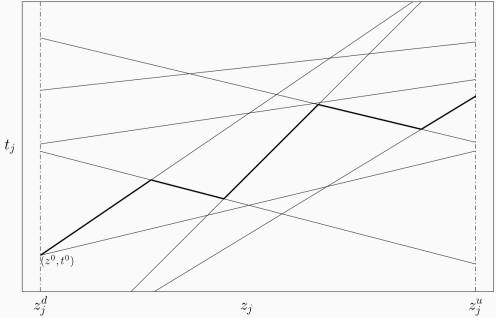

Rider Algorithm

Valid inequalities II: lines

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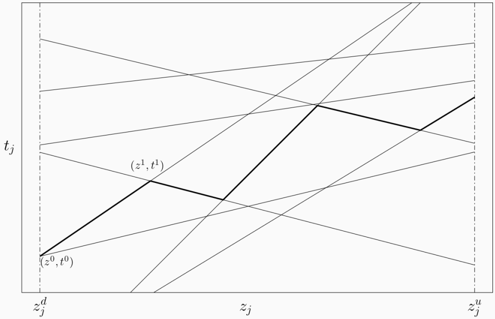

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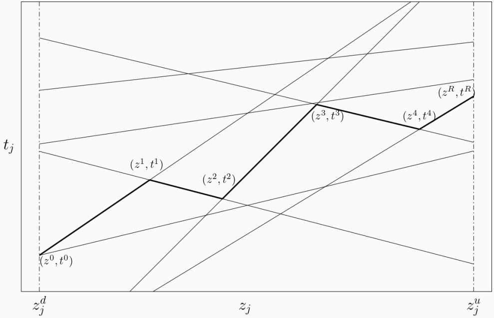

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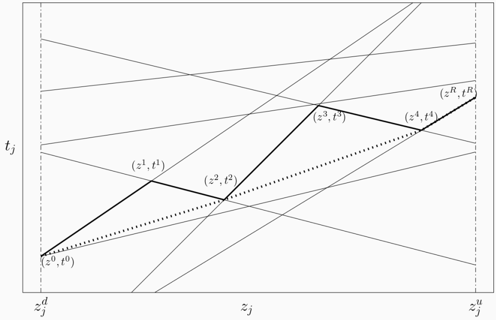

Lower hull: *Jarvis March*, *Graham scan*

Computational Results: OPF

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- Approaches **T**, **TS**, **V** and **TS+V** using five standard power systems.
- Instance: IEEE-118 test system: 118 nodes, 19 generators, 186 lines.
- GUROBI 9.1.2 on a Linux-based server with CPUs clocking at 2.6 GHz, 6 threads and 32 GB of RAM.
- 1000 scenarios, 5% violation of the JCC ($\epsilon = 0.05$, $p = 50$).
- Time limit: 10 hours.
- Results averaged over ten instances.

Computational Results

IEEE-118	**BN**	**T**(3)	**TS**(3)	**BN+V**	**TS**(1)+**V**
#CON	410413	100%	5.6%	101.6%	2.68%
LRgap	0.956%	0.434%	0.434%	0.4784%	0.2821%
MIPgap	0.29% (0)	0.12% (0)	0.01% (6)	0.03% (1)	0.00% (10)
Time	36000	1.0x	1.4x	1.1x	23.1x

Comparison

	Methods	IEEE-118
Average cost increase	**TS+V**	0.00%
	CVaR	0.57%
	ALSO-X	0.08%
	ALSO-X+	0.05%
Speedup factor	**TS+V**	23.1x
	CVaR	4045.5x
	ALSO-X	148.6x
	ALSO-X+	49.3x

The End

Á. Porras, C. Domínguez, J.M. Morales, and S. Pineda. (2022) Tight and compact sample average approximation for joint chance-constrained optimal power flow. *arXiv preprint arXiv:2205.03370*.

THANK YOU FOR YOUR ATTENTION