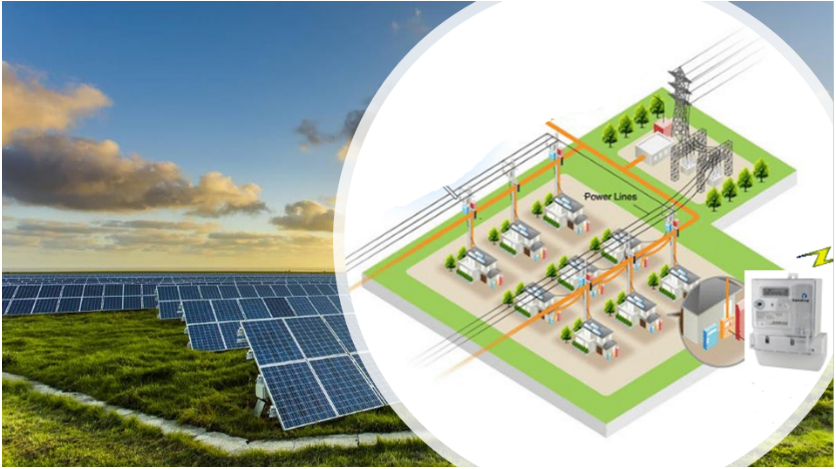
Depth-based Outlier Detection for Grouped Smart Meters: a Functional Data Analysis Toolbox

Antonio Elías, Juan Miguel Morales and Salvador Pineda


Power Lines

Outline

Depth-based Outlier Detection for Grouped Smart Meters:
a **Functional Data Analysis** Toolbox

1. Materials and Methods

   What is **Functional Data Analysis?**

   What is **Depth-based**?

2. Outlier detection toolbox

   Magnitude

   Shape

   Evolution

3. Numerical results

4. Conclusions

Materials and Methods

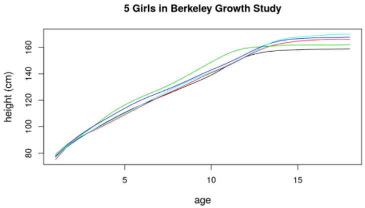
What is Functional Data Analysis?

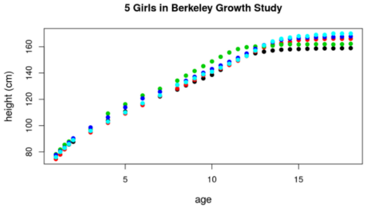
Functional Data Analysis (FDA) focuses on the analysis of smooth curves:

$$y_t(x), \quad x \in [a, b], \quad t = 1, \ldots, n$$

However, in practise we observe discrete evaluations of functional data samples:

$$y_t(\tilde{x}_j), \quad \tilde{x} \in [a, b], \quad t = 1, \ldots, T \quad j = 1, \ldots, p.$$





- Ramsay, J. O. and Silverman, B. W. (2005). Functional Data Analysis, Springer series in Statistics.
- Special Issues on Functional Data Analysis and Related Topics, Journal of Multivariate Analysis.

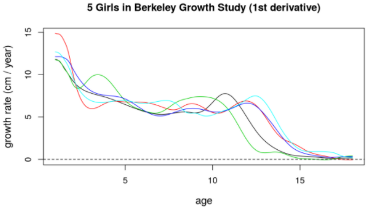
What is Functional Data Analysis?

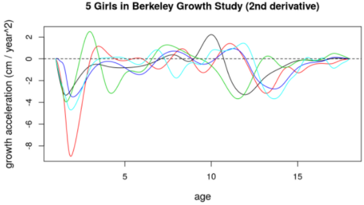
FDA propose to express discrete data by means of a basis expansion,

$$y_t(x) \approx \sum_{k=1}^{K} c_{t,k}\phi_k(x),$$

where $c_{t,k}$ are coefficients and ϕ_k a collection of basis functions with known derivatives

$$\partial_x^i y_t(x) \approx \sum_{k=1}^{K} c_{t,k}\partial_x^i \phi_k(x)$$





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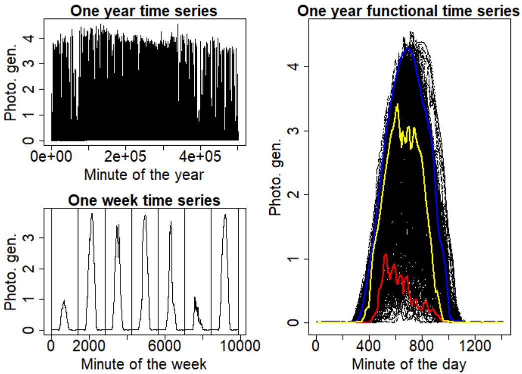
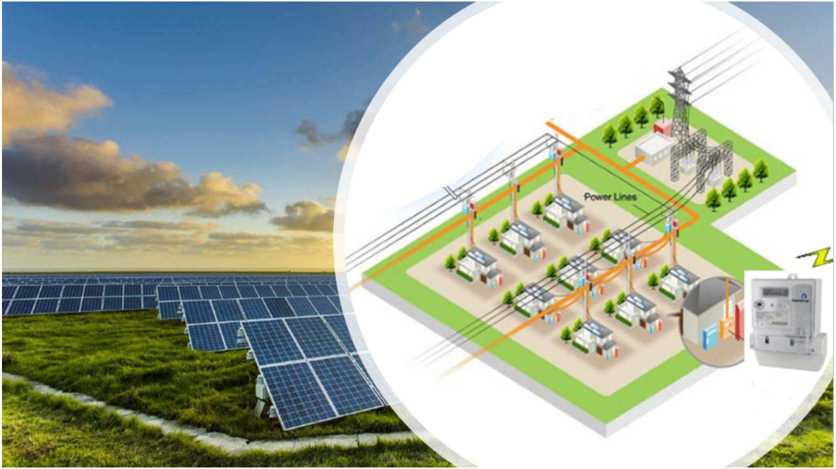
From one meter to FDA and Functional Time Series



Being again t be the index of days,

$$y_1(\tilde{x}), \ldots, y_T(\tilde{x}), \quad \tilde{x} = \{0, 1, \ldots, 1440\}, \quad t = \{1, \ldots, 365\}.$$

From multiple meters to High Dimensional Functional Time Series



From multiple meters to High Dimensional Functional Time Series

Let $i = 1, \dots, N$ be the index of the meters. Then, a sample of high dimensional functions takes the following form:

$$\mathbf{y}(x) = \begin{bmatrix} y_1^1(x) & y_1^2(x) & \dots & y_1^N(x) \\ y_2^1(x) & y_2^2(x) & \dots & y_2^N(x) \\ \vdots & \vdots & \ddots & \vdots \\ y_T^1(x) & y_T^2(x) & \dots & y_T^N(x) \end{bmatrix}.$$

We denote by $y_{\cdot}^i(x)$ the i-th column of $\mathbf{y}(x)$. This is the FTS of one smart meter that is composed of $y_1^i(x), \dots, y_T^i(x)$ daily functions.

In contrast, $y_t^{\cdot}(x)$ is the t-th row corresponding to N functions for a given day t. This is a sample of N daily functions $y_t^1(x), \dots, y_t^N(x)$ where each represents a meter.

Univariate data:

$$X : \Omega \to \mathbb{R}$$

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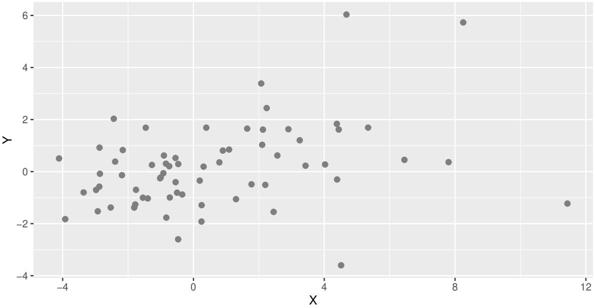
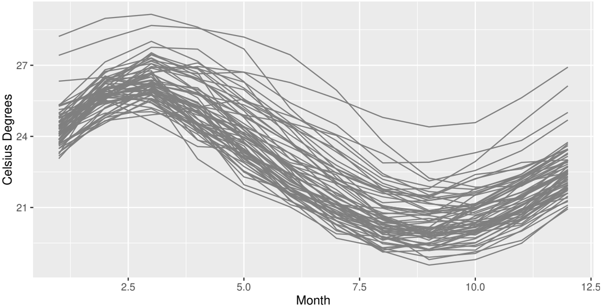
Bivariate data:

$$X = (X_1, X_2)^T : \Omega \to \mathbb{R}^2$$



Functional data:

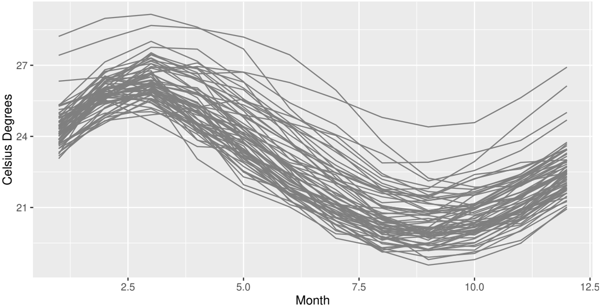
$$X(t) : t \in [a, b] \in \mathcal{C}[(a, b)]$$



What is Depth-based

Functional data:

$$X(t) : t \in [a, b] \in \mathcal{C}[(a, b)]$$



It is not straightforward! $\rightarrow$ **Depth Measures**

What is Depth-based

- It is a statistic that provides a real number for each datum that assigns its level of centrality.

$$\{\mathrm{FD}(y_t, P_T) = \mathrm{FD}(t), \quad t \in (1, \dots, T)\}.$$

- Depth-based is said to refer to a method that uses **depth measure**.

Romo, J. and Lopez-Pintado S. (2009). "On the concept of depth for functional data", JASA.

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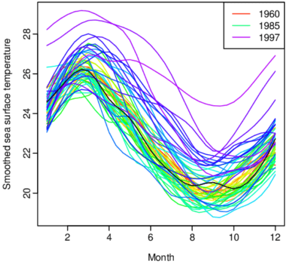

- We use the Modified Band Depth (MBD).

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Gijbels, I. and Nagy, S. (2017). "On a general definition of depth for functional data", Statist. Sci..

Outlier detection toolbox

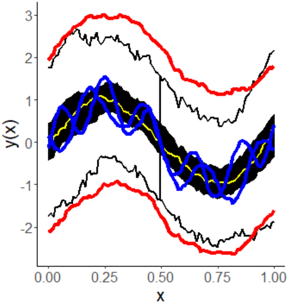
Outlier detection toolbox

We propose a toolbox based on the concept of depth to highlight:

- **Magnitude outliers**: the feature of analysis for the meter is higher or lower than the majority.
- **Shape outliers**: the shape of the profile for the meter is different than the majority.
- **Evolution outliers**: the day-to-day dependency for the meter is different for than the majority.

Magnitude Outlier

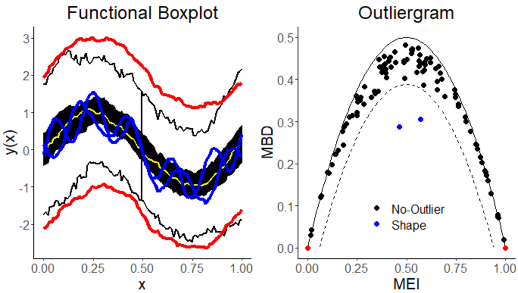
- **Functional Boxplot**. Sun, Y. and Genton, M. G. (2011). Functional boxplots. *Journal of Computational & Graphical Statistics*, 20(2):316–334.



- In the context of smart meters, we apply this method for every single day, where we have one curve for each meter.
- Then we determine as **magnitude outlier** those meters that have been unmasked as magnitude outlier the majority of the days.

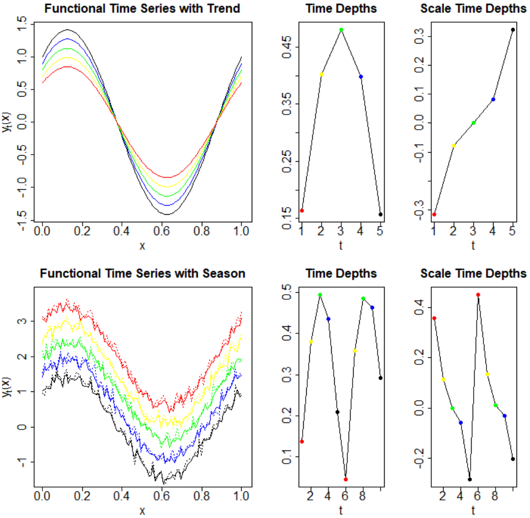
Shape Outlier

- **Outliergram**. Arribas-Gil, A. and Romo, J. (2014). Shape outlier detection and visualization for functional data: the outliergram. *Biostatistics*, 15(4):603–619.



- In the context of smart meters, we apply this method for every single day, where we have one curve for each meter.
- Then we determine as **shape outlier** those meters that have been unmasked as shape outlier the majority of the days.

Evolution Outlier



Evolution Outlier

- Lets define the time depths of each meter as,

$$\mathbf{FD}(t) = [\mathrm{FD}^1(t), \mathrm{FD}^2(t), \ldots, \mathrm{FD}^N(t)],$$

- **Prototype evolution** is a trimmed mean,

$$\mu\mathrm{FD}(t) = \frac{1}{\lceil \alpha N \rceil} \sum_{r=1}^{\lceil \alpha N \rceil} \mathrm{FD}^{[r]}(t),$$

- Then, we use the **Euclidean distance** between each $\mathbf{FD}(t)$ and the prototype $\mu\mathrm{FD}(t)$, i.e.,

$$d(\mathrm{FD}^i(t), \mu\mathrm{FD}(t)) = \sqrt{\sum_{t=1}^{T} \left(\mathrm{FD}^i(t) - \mu\mathrm{FD}(t)\right)^2},$$

- Right-tailed **outlier detection rule** for skewed distributions,

$$d(\mathrm{FD}^i(t), \mu\mathrm{FD}(t)) > Q_3(\mathrm{d}) + \gamma \times \exp^{3MC} \times IQR(\mathrm{d}).$$

Numerical results

Numerical results

- Pecan street data set.
- One-minute records of smart meters from Austin over one year.
- Voltage circuit and photo-voltaic energy generation.

		Zero derivative				First derivative			
	Meter id	**M**	**S**	**E**	$\widetilde{\mathbf{E}}$	**M**	**S**	**E**	$\widetilde{\mathbf{E}}$
Voltage	vol_{5746}			✓		✓			
	vol_{6139}			✓		✓			
	vol_{7901}			✓	✓	✓			
	vol_{9019}			✓					
	vol_{9922}					✓			
	vol_{7951}					✓			
Solar	sol_{9019}							✓	
	sol_{6139}							✓	
	sol_{3538}								✓

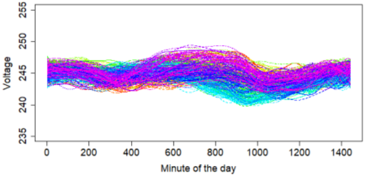
Key learnings

1 Evolution outliers are not detected by other methods.

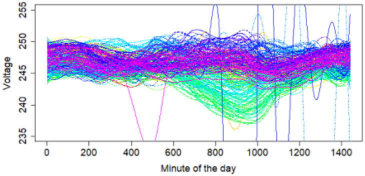
2 First derivatives allow detecting those outliers not detected with level data.

3 Scaled depths unmask those outliers which are not detected with regular depths.

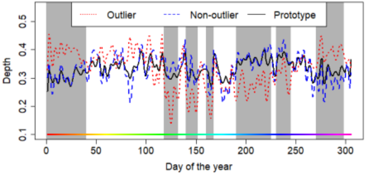
Household voltage circuit



(a) Non-outlier vol_{2818}

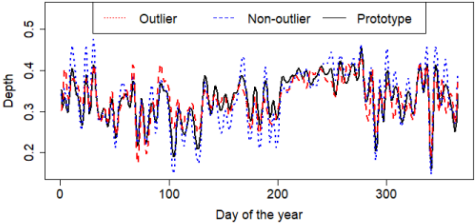


(b) Outlier vol_{9019}

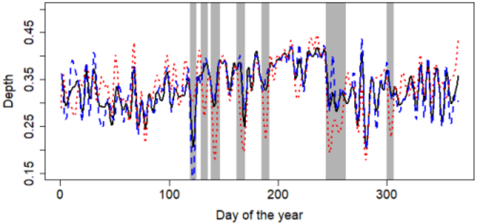


(c) Time series of depths

Household photo-voltaic energy generation

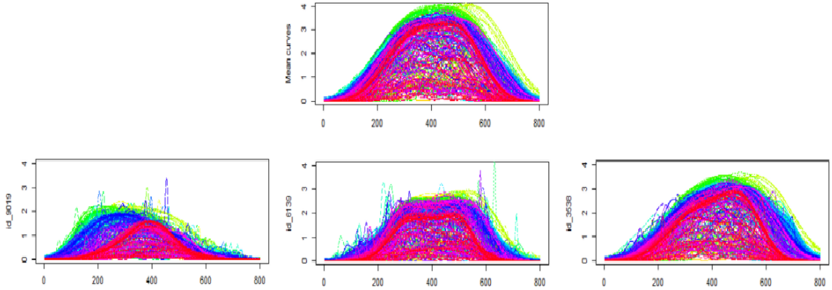


(a) Time series of depths computed on the zero derivative.



(b) Time series of depths computed on the first derivative.

Household photo-voltaic energy generation



Conclusions

Conclusions

- We propose a toolbox to highlight three types of outliers.
- Daily evolution is an important concern for outlier detection.
- Visit our website for more details `oasys.uma.es`.
  Article: `https://arxiv.org/abs/2107.01144`.
  Code: `smartOASYS` an R-package available at our Github Organization.
- Contact: `aelias@uma.es`.

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