Cost-aware Constraint Screening for the Unit Commitment Problem

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Joint work with:

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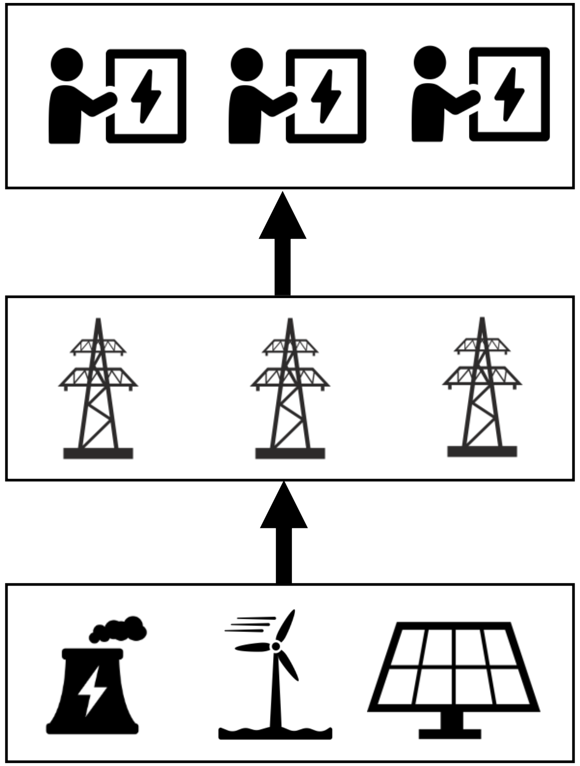
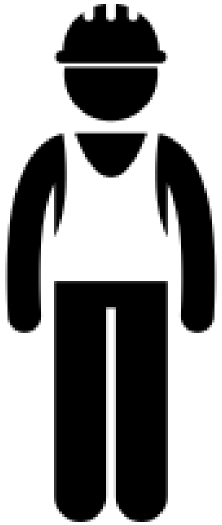
EURO ATHENS 2021 GREECE HYBRID

OASYS Optimization and Analytics for Sustainable energY Systems

Unit Commitment

- On/Off status
- Production level
- Physics and engineering constraints





$$\min_{\mathbf{x}\in\mathbb{R}^n,\,\mathbf{y}\in\{0,1\}^m} \quad f(\mathbf{x},\mathbf{y})$$

$$g_i(\mathbf{x},\mathbf{y}) \le 0, \quad \forall i$$

$$h_j(\mathbf{x}) \le 0, \quad \forall j$$

Large-scale Mixed-integer Program
NP-hard Problem

Unit Commitment

$$\min_{u_g, p_g, q_n} \sum_{g \in \mathcal{G}} c_g p_g$$

subject to:

$$q_n = \sum_{g \in \mathcal{G}_n} p_g - d_n, \quad \forall n \in \mathcal{N}$$

$$\sum_{n \in \mathcal{N}} q_n = 0$$

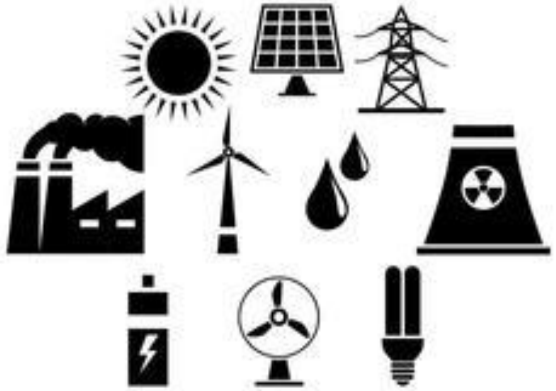
$$u_g \underline{p}_g \leq p_g \leq u_g \overline{p}_g, \quad \forall g \in \mathcal{G}$$

$$-\overline{f}_l \leq \sum_{n \in \mathcal{N}} a_{ln} q_n \leq \overline{f}_l, \quad \forall l \in \mathcal{L}$$

$$u_g \in \{0, 1\}, \quad \forall g \in \mathcal{G}$$

Simplifications:

- Single period.
- DC power flow.
- Known net demand.
- Linear costs.
- No failures.

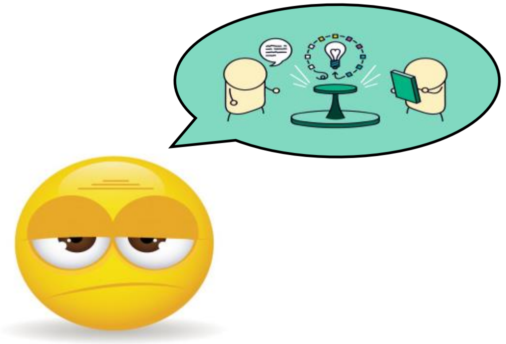


Eliminating Superfluous Transmission Constraints

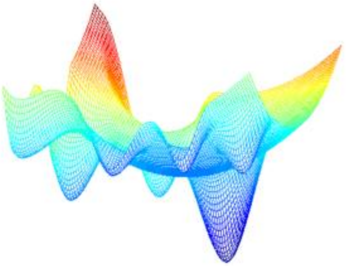
State of Art

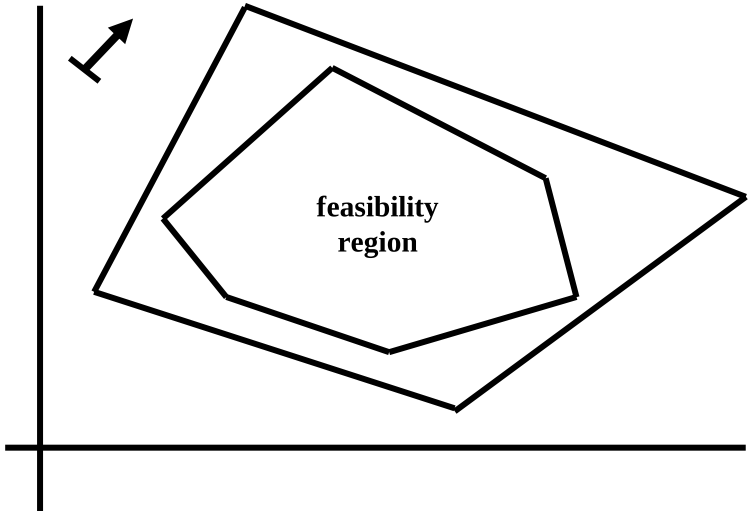
Machine Learning

Constraint Generation

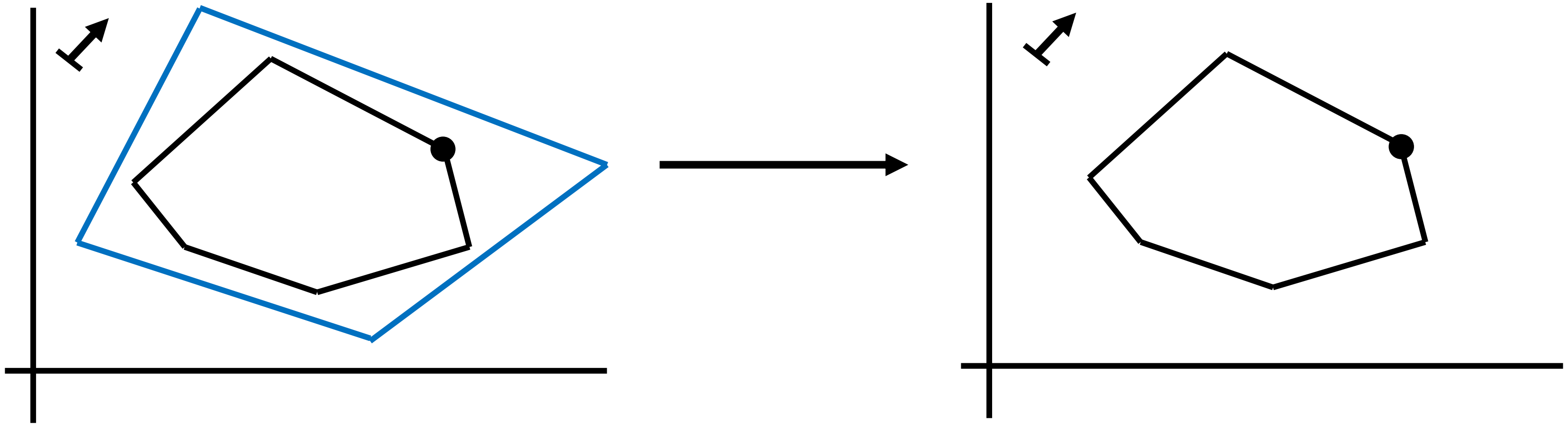


Optimization → Bounding Problem





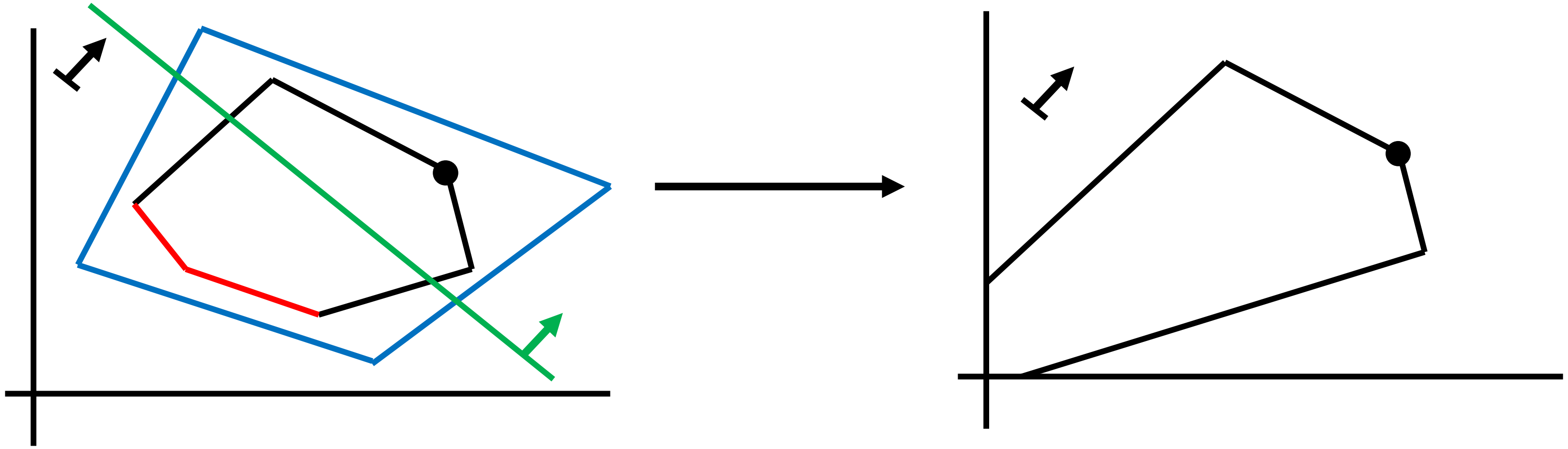
Current Methods



Legend

- Optimal Solution.
- Constraints not affecting the feasible region.
- Constraints not affecting the minimization of the objective function.
- Constraints with objective function information.

Proposed Approach



Current Methods:

- Reduce the original problem from 10 constraints to 6 constraints.

Proposed Approach:

- Reduce the original problem from 10 constraints to 4 constraints.

Screening Constraint Method

$$\max_{p_g, d_n, q_n} \;/\; \min_{p_g, d_n, q_n} \quad f_{l'} = \sum_{n \in \mathcal{N}} a_{l'n} q_n$$

subject to:

$$q_n = \sum_{g \in \mathcal{G}_n} p_g - d_n, \quad \forall n \in \mathcal{N}$$

$$\sum_{n \in \mathcal{N}} q_n = 0$$

$$0 \leq p_g \leq \overline{p}_g, \quad \forall g \in \mathcal{G}$$

$$-\overline{f}_l \leq \sum_{n \in \mathcal{N}} a_{ln} q_n \leq \overline{f}_l, \forall l \in \mathcal{L}, l \neq l'$$

$$\mathbf{d} \in \mathcal{D}$$

L. A. Roald and D. K. Molzahn, "Implied Constraint Satisfaction in Power System optimization: The Impacts of Load Variations," in *2019 57th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pp. 308–315, 2019.

$$\boxed{\underline{d}_n \leq d_n \leq \overline{d}_n}$$

When $f_{l'}$ is minimized …

- If $-\overline{f_{l'}} \leq f_{l'}$ → the constraint can be removed.

When $f_{l'}$ is maximized …

- If $f_{l'} \leq \overline{f_{l'}}$ → the constraint can be removed.

Conservative approach

Non-economical dispatches.
Net demands without spatial correlation.

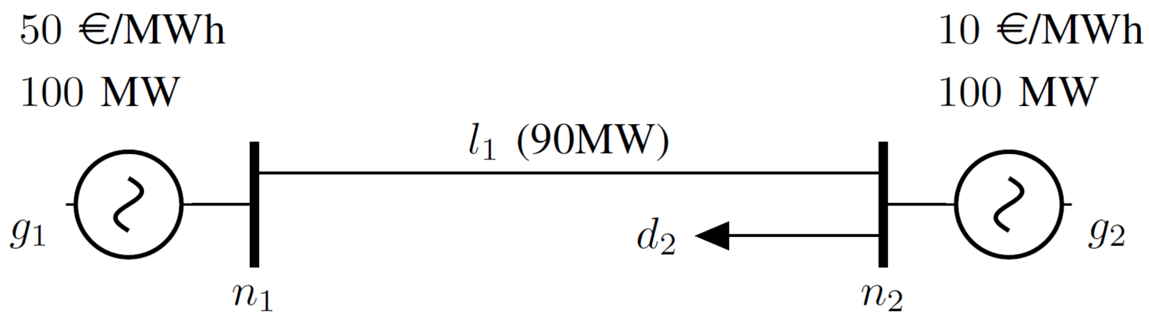
Proposal

Impose a maximum cost to the operation of generators.

$$\sum c_g p_g \leq \bar{C}$$

Net demand as convex combination of observed instances.

$$d_n = \sum \alpha_n \hat{d}_n \qquad \sum \alpha_n = 1$$



$$\max_{p_g,d_n,q_n} \Big/ \min_{p_g,d_n,q_n} \quad f_{l'} = \sum_{n\in\mathcal{N}} a_{l'n} q_n$$

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$$\mathbf{d} \in \mathcal{D}$$

+

$$\boxed{\sum_{g\in\mathcal{G}} c_g p_g \le \overline{C}}$$

$\boxed{80 \le d \le 120}$ $\boxed{-90 \le f_1 \le 90}$

Solution to the Min problem $\rightarrow$ $f_1 = 0\ MW$

Solution to the Max problem $\rightarrow$ $f_1 = 100\ MW$

$-\overline{f_1} \le f_1$ is eliminated

$f_1 \le \overline{f_1}$ is retained

$$\left.\begin{array}{l} P_1 = 20\ MW \\ P_2 = 100\ MW \\ f_1 = 20\ MW \end{array}\right\} \quad Cost = 2000\ €$$

$$50 \cdot P_1 + 10 \cdot P_2 \le 2000$$

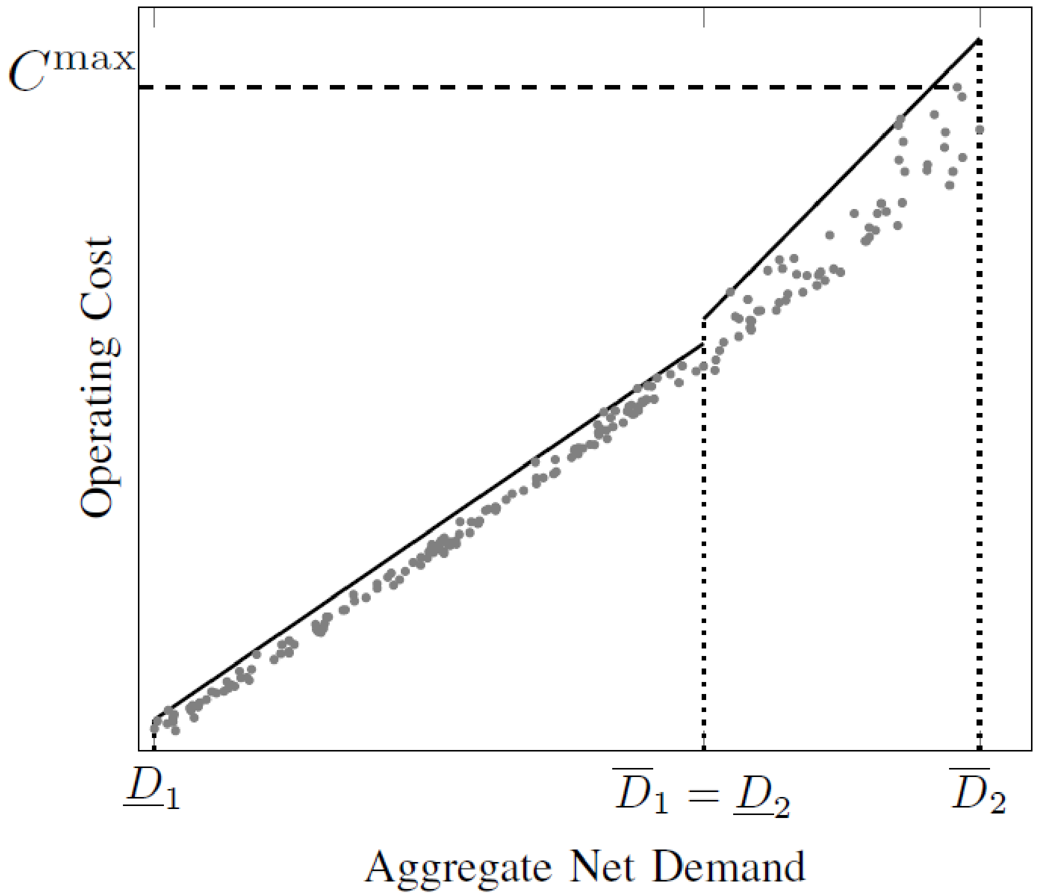
$$\downarrow$$

$P_1 \le 40$

$-\overline{f_1} \le f_1$ is eliminated

$f_1 \le \overline{f_1}$ is eliminated

$$\boxed{\sum_{g\in\mathcal{G}} c_g p_g \leq \overline{C}} \begin{cases} \overline{C} = C^{\max} \\ \\ \overline{C} = \overline{C}(\mathbf{d}) \end{cases}$$



Modeling

- Aggregate net demand
- Piecewise linear function
- Quantile regression in each segment

VS

Comparison

$$\max_{p_g,d_n,q_n} / \min_{p_g,d_n,q_n} \quad f_{l'} = \sum_{n\in\mathcal{N}} a_{l'n} q_n$$

subject to:

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$$\mathbf{d} \in \mathcal{D}$$

Methods:

- **BN:** Benchmark method.
- **UB:** BN + imposing a maximum cost for the operation of the power plants.
- **CC:** Net demand as a convex combination of past instances.
- **UB+CC:** CC + the economical information.

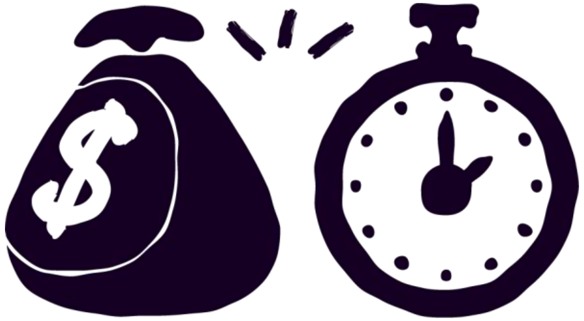
Case Study

Description of the procedure:

1. Use historical information on past unit commitment instances
2. Determine the set of line capacity constraints that can be removed from the original UC problem
3. Solve the reduced unit commitment problem
4. Fix the binary commitment decisions

All methods obtain the same optimal solution as the original Unit Commitment

Case Study



Results:

- **UB** does not achieve significant results due to spatial correlations of net nodal demands.
- **CC** gets a meaningful reduction with respect to previous methods.
- **UC+CC** achieves significant results decreasing the retained constraints and the computation burden by 15% and 28%, respectively.

2000 bus-system

8640 hours of past unit commitment instances

- **7200 hours as training set**
- **1440 hours as test set**

	BN	UB	CC	UB+CC
Retained constraints (%)	33.8	33.0	24.9	18.9
Computational burden (%)	50.7	48.7	35.4	22.5

Conclusions

- ✓ We screen out constraints not affecting both the feasibility region and the minimization of the objective function.
- ✓ Our proposal gets a reduction in terms of retained constraints and computational burden by 15% and 28%, respectively.
- ✓ We reduce the computational time of the original Unit Commitment problem up to 77.5%.

Preprint available
https://arxiv.org/abs/2104.05746

Thank you for your attention

Any questions?

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http://oasys.uma.es/

https://groupoasysflexanalytics.readthedocs.io/en/latest/

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