A bilevel framework for decision-making under uncertainty with contextual information

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July 2021

Decision under uncertainty: ice cream shop

- At 10 am we have to decide how much ice cream to make (decision z)
- At 10 am we do not know the demand in the afternoon (uncertain parameter y)
- We can use some available information such as the temperature at 10 am (contextual data x)
- Obviously there is a relationship between the morning temperature (x) and the ice cream demand in the afternoon (y).
- We would like to use such relation to make **better decisions** about ice cream quantity (z)

$$\min_{z \in Z} \mathbb{E}[f_0(z; y) | X = x]$$

Decision under uncertainty: some approaches

$$\min_{z \in Z} \mathbb{E}[f_0(z; y) | X = x]$$

x contextual info

y uncertain parameter

z decision

Approach FO

1) $$w^{\mathrm{FO}} = \arg\min_{w \in \mathbb{R}^q} \sum_{i \in \mathcal{N}} (y_i - w^T x_i)^2$$

2) $$\hat{y} = w^{\mathrm{FO}} x$$

$$z^{\mathrm{FO}} = \arg\min_{z \in Z} f_0(z; \hat{y})$$

Approach DR [1]

1) $$w^{\mathrm{DR}} = \arg\min_{\hat{z} \in Z} \sum_{i \in \mathcal{N}} f_0(\hat{z}, y_i)$$

$$s.t. \;\; \hat{z} = w^T x_i$$

2) $$z^{\mathrm{DR}} = w^{\mathrm{DR}} x$$

[1] BAN, Gah-Yi; RUDIN, Cynthia. The big data newsvendor: Practical insights from machine learning. *Operations Research*, 2019, vol. 67, no 1, p. 90-108.

Cooking a new data-driven approach

Considering the dataset $S = \{(x_i, y_i), \forall i \in \mathcal{N}\}$

Taking into account the nominal objective function $f_0(z; y)$

Can we find a better (less costlier) estimate? $\hat{y} = w^T x$

We can try iteratively:

Grid search on the parameters: w

Compute the estimate for the training set: $\hat{y}_i = w^T x_i, \ \ \forall i \in \mathcal{N}$

Compute the optimal decision for the training set: $z_i = \min_{z \in Z} f_0(z; \hat{y}_i), \ \ \forall i \in \mathcal{N}$

Compute the total cost of those parameters: $\sum_{i \in \mathcal{N}} f_0(z_i; y_i)$

Choose the parameter w with minimum total cost

Bilevel approach

Bilevel approach (BL)

(C) 1) $$w^{\mathrm{BL}} = \arg\min_{w \in \mathbb{R}^q} \sum_{i \in \mathcal{N}} f_0(\hat{z}_i; y_i)$$

(B) $$\text{s.t.} \quad \hat{z}_i = \arg\min_{z \in Z} f_0(z; \hat{y}_i)$$

(A) $$\text{s.t.} \quad \hat{y}_i = g(x; w^{\mathrm{BL}})$$

$$\forall i \in \mathcal{N}$$

2) $$\hat{y} = g(x; w^{\mathrm{BL}})$$

$$z^{\mathrm{BL}} = \arg\min_{z \in Z} \; f_0(z; g^{\mathrm{BL}}(x; \hat{y}))$$

(A) Compute the estimate for the training set

(B) Compute the optimal decision for the training set

(C) Compute the total cost of parameters

$$g^{\mathrm{BL}}(x; w) = w^T x$$

linear case

Taming the wild bilevel (I)

$$w^{\mathrm{BL}} = \arg\min_{w \in \mathbb{R}^q} \sum_{i \in \mathcal{N}} f_0(\hat{z}_i; y_i)$$

Lower Level

$$\text{s.t. } \hat{z}_i = \arg\min_z f_0(z; g^{\mathrm{BL}}(x_i; w)) \ \ \forall i \in \mathcal{N}$$

$$\left.\begin{aligned} f_j(z_i) &\leq 0, \ \ \forall j \\ h_k(z_i) &= 0, \ \ \forall k \end{aligned}\right\} \ \forall i \in \mathcal{N}$$

Feasible set Z

Under convexity assumptions

KKT optimality conditions

$$w^{\mathrm{BL}} = \arg\min_{w, \hat{z}_i, \lambda_{ji}, \upsilon_{ki}} \sum_{i \in \mathcal{N}} f_0(\hat{z}_i; y_i)$$

$$\text{s.t. } \nabla f_0(\hat{z}_t, g^{\mathrm{BL}}(x_t, w)) + \sum_{j \in \mathcal{J}} \lambda_{ji} \nabla f_j(\hat{z}_i) + \sum_{k \in \mathcal{K}} \upsilon_{ki} \nabla h_k(\hat{z}_i) = 0, \ \ \forall i \in \mathcal{N}$$

(stationarity)

$$\left.\begin{aligned} f_j(\hat{z}_i) &\leq 0, \ \ \forall j \\ h_k(\hat{z}_i) &= 0, \ \ \forall k \\ \lambda_{ji} &\geq 0, \ \ \forall j \\ \lambda_{ji} f_j(\hat{z}_i) &= 0, \ \ \forall j \end{aligned}\right\} \ \forall i \in \mathcal{N}$$

(primal feasibility)

(dual feasibility)

(complementary slackness)

Single level optimization problem

Taming the wild bilevel (II)

$$\lambda_{ji} f_j(\hat{z}_i) = 0, \quad \forall j, \quad \forall i \in \mathcal{N}$$

(complementary slackness)

BL-R Approach: Regularization

$$-\sum_{\forall ji} \lambda_{ji} f_i(\hat{z}_i) \leq \epsilon$$

NLP

CONOPT

Iteratively solved for:

$\epsilon \geq 0, \quad \epsilon \to 0$

↑ fast, ↓ non-optimal

BL-M Approach: Big-M

$$\left.\begin{array}{l} \lambda_{ji} \leq u_{ji} M^D \\ f_i(\hat{z}_i) \geq (u_{ji} - 1) M^P \\ u_{ji} \in \{0, 1\} \end{array}\right\} \forall j, \forall i \in \mathcal{N}$$

MIQP

CPLEX

↑ optimality, ↓ slow, big-M tunning

Decision under uncertainty (I)

$$z^* = \arg\min_{z \in Z} \mathbb{E}[f_0(z;y)|X=x]$$

$$S = \{(x_i, y_i), \forall i \in \mathcal{N}\}$$

Benchmark method (BN)

$$z^{\mathrm{BN}} = \arg\min_{z} f_0(z;y)$$

Perfect knowledge of y

Predictive approach (FO)

$$w^{\mathrm{FO}} = \arg\min_{w \in \mathbb{R}^q} \sum_{i \in \mathcal{N}} l^{\mathrm{FO}}(g^{\mathrm{FO}}(x_i;w), y_i)$$

$$z^{\mathrm{FO}} = \arg\min_{z \in Z} f_0(z; g^{\mathrm{FO}}(x; w^{\mathrm{FO}}))$$

Bilevel approach (BL)

$$\begin{aligned} w^{\mathrm{BL}} = \arg\min_{w \in \mathbb{R}^q} & \sum_{i \in \mathcal{N}} f_0(\hat{z}_i; y_i) \\ \text{s.t. } & \hat{z}_i = \arg\min_{z \in Z} f_0(z; g^{\mathrm{BL}}(x_i; w)), \quad \forall i \in \mathcal{N} \end{aligned}$$

$$z^{\mathrm{BL}} = \arg\min_{z \in Z} f_0(z; g^{\mathrm{BL}}(x; w^{\mathrm{BL}}))$$

Decision Rule approach (DR[1])

$$\begin{aligned} w^{\mathrm{DR}} = \arg\min_{w \in \mathbb{R}^q} & \sum_{i \in \mathcal{N}} f_0(g^{\mathrm{DR}}(x_i; w); y_i) \\ \text{s.t. } & g^{\mathrm{DR}}(x_i; w) \in Z, \quad \forall i \in \mathcal{N} \end{aligned}$$

$$z^{\mathrm{DR}} = g^{\mathrm{DR}}(x; w^{\mathrm{DR}})$$

[1] BAN, Gah-Yi; RUDIN, Cynthia. The big data newsvendor: Practical insights from machine learning. *Operations Research*, 2019, vol. 67, no 1, p. 90-108.

Decision under uncertainty (II)

$$z^* = \arg\min_{z \in Z} \mathbb{E}[f_0(z; y) | X = x] \qquad S = \{(x_i, y_i), \forall i \in \mathcal{N}\}$$

- Forecasting approach (FO)

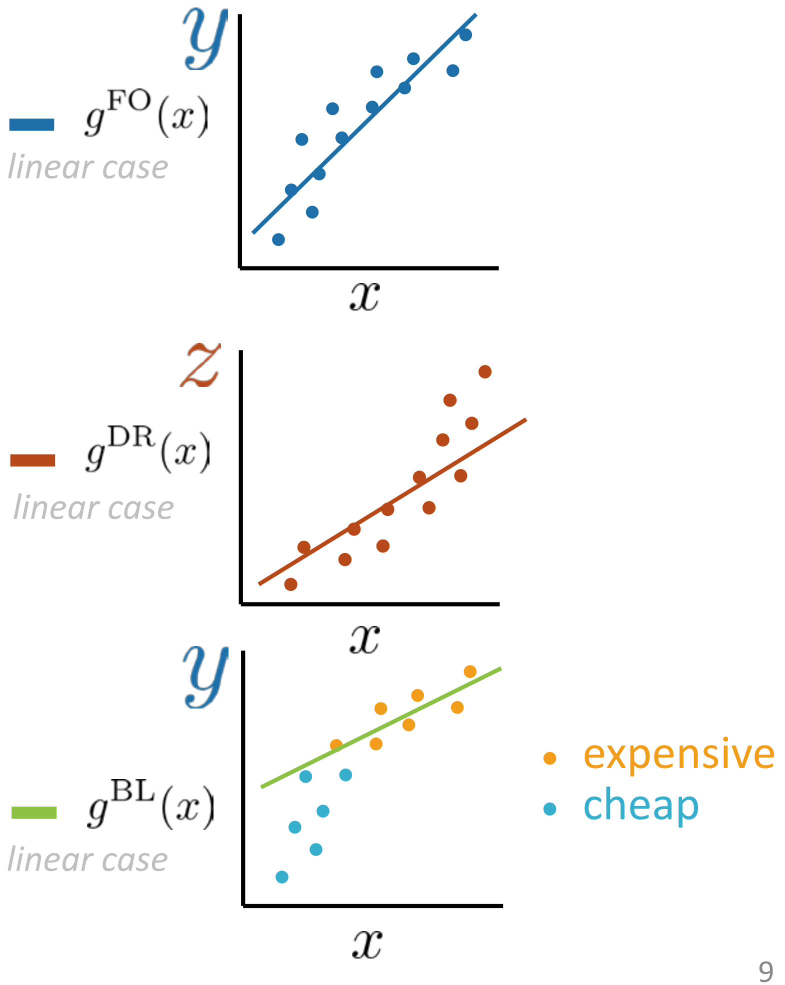
  learns the relation between y and x ignoring f_0 and Z

- Decision rule approach (DR)

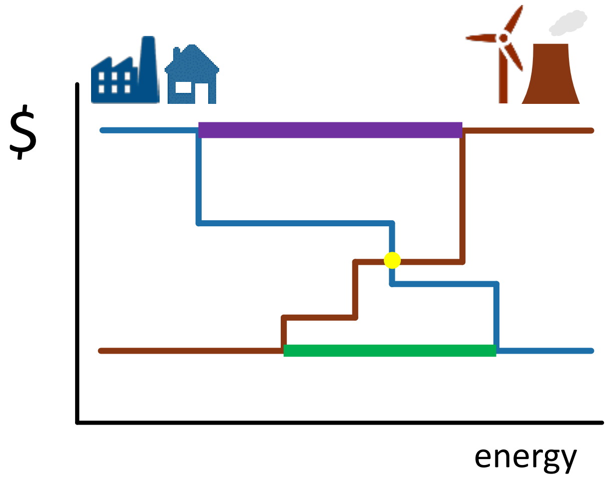
  learns the relation between z^* and x

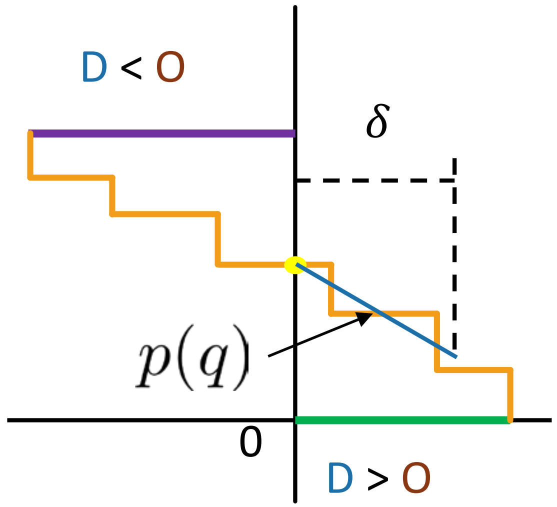
- Bilevel approach (BL)

  learns the relation between y and x taking into account f_0 and Z



Case study: Strategic producer





$$q^* = \arg \max_{\underline{q} \leq q \leq \overline{q}} \; p(q)q - c(q)$$

$$\begin{cases} p(q) & = \alpha - \beta q \\ c(q) & = c_2 q^2 + c_1 q + c_0 \end{cases} \qquad \begin{cases} \beta' = \beta + c_2 \\ \alpha' = \alpha - c_1 \end{cases}$$

$$\boxed{q^* = \arg \min_{\underline{q} \leq q \leq \overline{q}} \; \beta' q^2 - \alpha' q}$$

Case study assumptions:

- Our behaviour does not influence the rest of the competitors.
- Our offer is just a single quantity.
- The transmission grid is neglected.
- A linear function can approximate the residual demand.

Case study: Method particularization

Approach BN

$$q_i^{\mathrm{BN}} = \arg \min_{\underline{q} \leq q \leq \overline{q}} \; \beta_i' q^2 - \alpha_i' q$$

$$g(x; w) = w^T x$$

$$\begin{cases} q_i & = \arg\min_{\underline{q} \leq q \leq \overline{q}} \; q^2 - \gamma_i q \\ \gamma_i & = \alpha_i' / \beta_i', \;\; \beta_i' > 0 \end{cases}$$

Approach FO

$$w_\alpha^{\mathrm{FO}} = \arg \min_{w \in \mathbb{R}^q} \sum_{i \in \mathcal{N}} (\alpha_i' - w^T x_i)^2$$

$$w_\beta^{\mathrm{FO}} = \arg \min_{w \in \mathbb{R}^q} \sum_{i \in \mathcal{N}} (\beta_i' - w^T x_i)^2$$

$$q^{\mathrm{FO}} = \arg \min_{\underline{q} \leq q \leq \overline{q}} \hat{\beta}' \;\; q^2 - \hat{\alpha}' \;\; q$$

Approach BL

$$w^{\mathrm{BL}} = \arg \min_{w \in \mathbb{R}^q} \sum_{i \in \mathcal{N}} \beta_i' \hat{q}_i^2 - \alpha_i' \hat{q}_i$$

$$\text{s.t.} \;\; \hat{q}_t = \arg \min_{\underline{q} \leq q \leq \overline{q}} q^2 - w^T x_i q, \;\; \forall i \in \mathcal{N}$$

$$q^{\mathrm{BL}} = \arg \min_{\underline{q} \leq q \leq \overline{q}} \;\; q^2 - (w^{\mathrm{BL}})^T x \;\; q$$

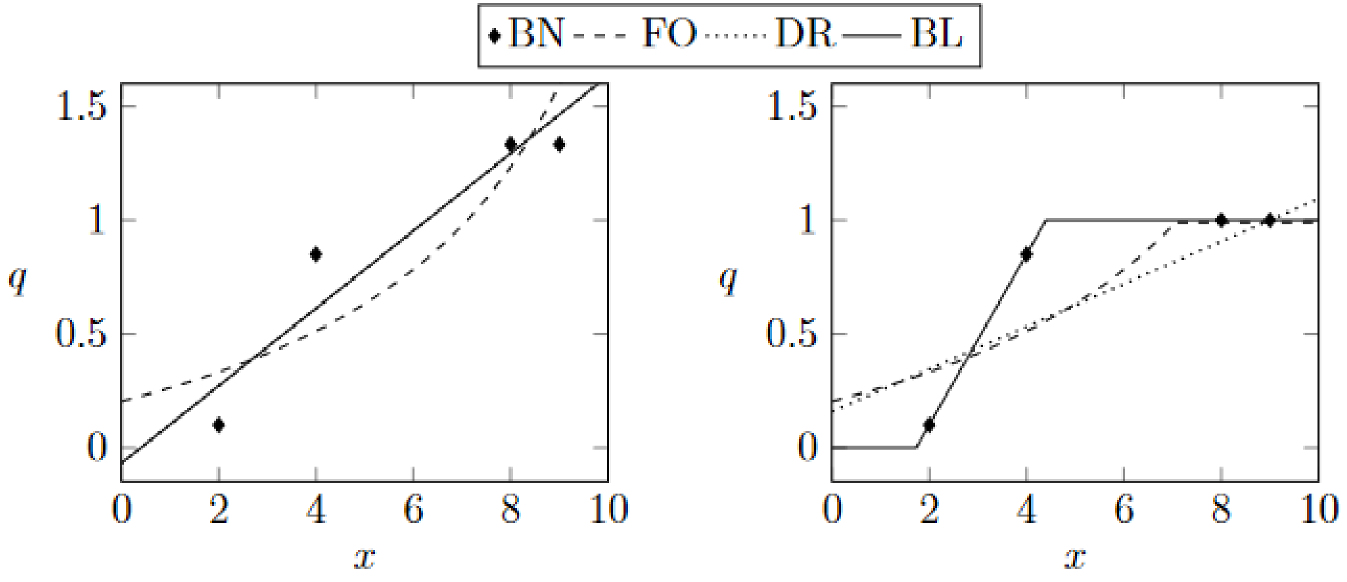
Approach DR

$$w^{\mathrm{DR}} = \arg \min_{w \in \mathbb{R}^q} \sum_{i \in \mathcal{N}} \beta_i' (w^T x_i)^2 - \alpha_i' w^T x_i$$

$$\text{s.t.} \;\; \underline{q} \leq w^T x_i \leq \overline{q}, \;\; \forall i \in \mathcal{N}$$

$$q^{\mathrm{DR}} = (w^{\mathrm{DR}})^T x$$

Case study: An illustrative example

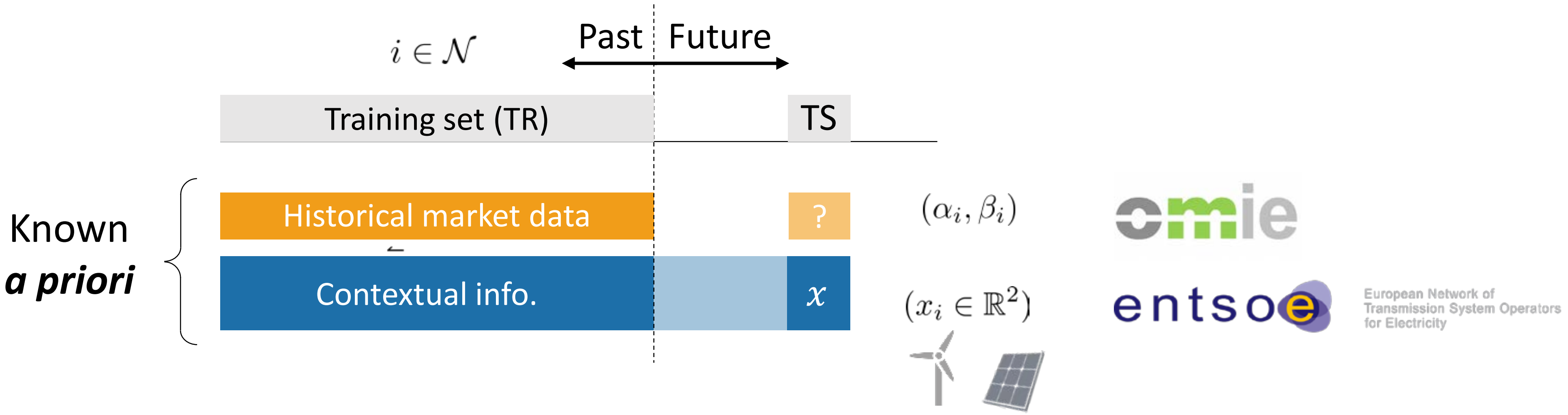


$$q_i^{\mathrm{BN}} = \arg \min_{0 \leq q \leq 1} \beta_i' q^2 - \alpha_i' q$$

RI (%)		
	(a)	(b)
BN	100.0	100.0
FO	91.0	92.5
DR	95.9	91.8
BL	95.9	100.0

- DR and BL coincide in the unconstrained case (in this example).
- DR deliver infeasible solutions for x > 9 (constrained case).
- BL outperform the other approaches.

Case study: Realistic case study setup



Experimental setup

- Real data from Iberian electricity market is used to approximate the inverse demand function.
- Contextual info: Wind and Solar power production forecast $(x_i \in \mathbb{R}^2)$
- Three different generation technologies: base (nuclear), medium (carbon) and peak (gas).
- 43 Bins of 200 points, 80% training, 20% test to compute average results.

Case study: Numerical results (I)

	c_1 (€/MWh)	$\overline{q}$ (MW)
Base	10	1000
Medium	35	500
Peak	50	250

$c_2 = 0.005$ €/MWh^2

	Base	Medium	Peak
Relative income FO	96.0%	77.3%	41.6%
Relative income DR	94.6%	62.6%	18.9%
Relative income BL	96.3%	80.0%	58.7%
Infeasible cases DR	4.9%	1.7%	0.1 %

- All methods provide similar incomes for the base unit.
- The uncertainty significantly affects medium and peak units.
- The proposed BL approach obtains the highest incomes
- DR approach lead to a significant number of infeasible cases.

Case study: Numerical results (II)

	$RI^{\text{BL-R}}$	$RI^{\text{BL-M}}$
Base	96.3%	96.3%
Medium	79.2%	80.0%
Peak	58.4%	58.7%

Performance BL-R vs BL-M

	FO	DR	BL-R	BL-M
Base	0.24	0.65	3.90	197.77
Medium	0.35	1.06	6.80	149.89
Peak	0.26	0.78	4.62	22.68

Average time (s) for different methods

- BL-R: NLP (CONOPT)
- BL-M: MIQP (CPLEX)

- BL-R used to initialized BL-M.
- BL-M achieves better results but it is computationally expensive.
- BL-R provides a fast and good solutions.

Methods Summary

Forecasting approach (FO)

- Learns the relation between y and x ignoring f_0 and Z.
- Wide variety of learning techniques can be applied.
- Obtained decisions may be suboptimal.

Decision rule approach (DR)

- Learns the relation between z^* and x.
- Decisions are quickly obtained without solving an optimization problem.
- Obtained decisions may be infeasible.

Bilevel approach (BL)

- Learns the relation between y and x taking into account f_0 and Z.
- Best possible decisions using available contextual information.
- Bilevel problem can be only solved under certain assumptions.

Conclusions

- Novel data-driven framework for conditional stochastic optimization, where parameters are formulated as a function of some contextual information.

- Application to the problem of a strategic producer supplying the residual demand in a day ahead electricity market with realistic data.

- Numerical experiments show our proposal can significantly increase the performance of the strategic producer.

THANKS!

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Checkout more at:

M. A. Muñoz, S. Pineda and J. M. Morales, *"A bilevel framework for decision-making under uncertainty with contextual information," arXiv preprint arXiv:2008.01500*

Acknowledgement: M. Á. Muñoz is funded by the Spanish Ministry of Science, Innovation and Universities through the State Training Subprogram 2018 of the State Program for the Promotion of Talent and its Employability in R&D&I, within the framework of the State Plan for Scientific and Technical Research and Innovation 2017-2020 and by the European Social Fund.