A homothetic inverse optimization approach to forecast the price-response of a pool of buildings

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OASYS

Optimization and Analytics for Sustainable energY Systems

Context and motivation

- Buildings may profit from their **thermal capacity** by
  - shifting their load
  - reducing their peak demand

  in response to power system/market conditions ($\equiv$ **electricity price**).

- A set of thermostatically-controlled buildings aiming to minimize the electricity bill while ensuring users' comfort.

Goal

- Modeling and forecasting the **aggregate price-response** of the ensemble of buildings.
- Key to power system and market operations.
- Principal challenges:
  - Forecast lead time: One day ahead.
  - Buildings thermal dynamics.
  - Occupants' comfort.

Key ingredients of our approach

- **Building prototype**. Represents the *average* behavior of the buildings in the pool. Notation: "$\square^p$".
- **Homothety**. The feasible operational region of the pool of buildings ("$\square^a$") as a homothet of that of the building prototype.
  - Dilation factor β.
  - Translation vector $\boldsymbol{\tau}$.
- **(Piece-wise linear) utility function.** Maximization of welfare and minimization of discomfort costs. Vector of marginal utilities $\mathbf{m}$.
- **Inverse optimization (IO)**. Inference of β, $\boldsymbol{\tau}$ and $\mathbf{m}$: input parameters of an optimization problem (i.e., the forecasting model).

Building prototype: Feasible region

$$\theta_h^p = a_1\theta_{h-1}^p + (1 - a_1)\left[\theta_h^{amb} - a_2 p_h^p\right], \quad \forall h \in \mathcal{H} \tag{1a}$$

$$\underline{\theta}_h^p \le \theta_h^p \le \overline{\theta}_h^p, \quad \forall h \in \mathcal{H} \tag{1b}$$

$$0 \le p_h^p \le P, \quad \forall h \in \mathcal{H} \tag{1c}$$

- θ_h: Indoor temperature in time period h, comfort bounds $\underline{\theta}_h$, $\overline{\theta}_h$.
- p_h: Cooling power, rated power P.
- θ_h^{amb}: Ambient temperature in time period h.
- a_1, a_2: Building parameters (depending on thermal resistance and capacitance, COP and discretization time step).

Building prototype: Feasible region in matrix form

$$\mathbf{0} \leq \boldsymbol{p}^p \leq \mathbf{P} \quad (= P \cdot \mathbf{1}_{|\mathcal{H}|}) \tag{2a}$$

$$\underline{\boldsymbol{\theta}}^p \leq \boldsymbol{\Lambda} \boldsymbol{B} \boldsymbol{p}^p + \boldsymbol{\Lambda}(\boldsymbol{c}^p + \boldsymbol{t}^p) \leq \overline{\boldsymbol{\theta}}^p \tag{2b}$$

- $\boldsymbol{c}^p = [a_1 \theta_0^p, 0, \ldots, 0]^T$: Vector of initial conditions.
- $\boldsymbol{t}^p = \boldsymbol{\theta}^{amb}(1 - a_1)$.

Aggregate building model: Feasible region in matrix form

$$\mathbf{0} \leq \boldsymbol{p}^p \leq \mathbf{P} \;\; (= P \cdot \mathbf{1}_{|\mathcal{H}|}) \tag{2a}$$

$$\underline{\boldsymbol{\theta}}^p \leq \boldsymbol{\Lambda B p}^p + \boldsymbol{\Lambda}(\boldsymbol{c}^p + \boldsymbol{t}^p) \leq \overline{\boldsymbol{\theta}}^p \tag{2b}$$

Homothety: $\boldsymbol{p}^a = \beta \boldsymbol{p}^p + \boldsymbol{\tau}$

$$\boldsymbol{\tau} \leq \boldsymbol{p}^a \leq \beta \mathbf{P} + \boldsymbol{\tau} \tag{3a}$$

$$\beta \underline{\boldsymbol{\theta}}^p + \boldsymbol{\Lambda B \tau} \leq \boldsymbol{\Lambda B p}^a + \boldsymbol{\Lambda} \beta \left(\boldsymbol{c}^p + \boldsymbol{t}^p \right) \leq \beta \overline{\boldsymbol{\theta}}^p + \boldsymbol{\Lambda B \tau}. \tag{3b}$$

Forecasting model

Forecast for day d, given the price vector $\boldsymbol{\lambda}_d$ and discomfort costs $\mathbf{c}^s$:

$$\max_{\boldsymbol{p}^a_{b,d}, \boldsymbol{s}^a_d} \quad \sum_{b \in \mathcal{B}} \left(\boldsymbol{m}_{b,d} - \boldsymbol{\lambda}_d\right)^T \boldsymbol{p}^a_{b,d} - \left(\boldsymbol{c}^s\right)^T \boldsymbol{s}^a_d \tag{4a}$$

subject to:

$$\boldsymbol{\tau} \leq \sum_{b \in \mathcal{B}} \boldsymbol{p}^a_{b,d} \leq \beta \mathbf{P} + \boldsymbol{\tau} \tag{4b}$$

$$\beta \underline{\boldsymbol{\theta}}^p_d + \boldsymbol{\Lambda} \boldsymbol{B} \boldsymbol{\tau} - \boldsymbol{s}^a_d \leq \sum_{b \in \mathcal{B}} \boldsymbol{\Lambda} \boldsymbol{B} \boldsymbol{p}^a_{b,d} + \boldsymbol{\Lambda} \beta \left(\boldsymbol{c}^p_d + \boldsymbol{t}^p_d\right) \tag{4c}$$

$$\sum_{b \in \mathcal{B}} \boldsymbol{\Lambda} \boldsymbol{B} \boldsymbol{p}^a_{b,d} + \boldsymbol{\Lambda} \beta \left(\boldsymbol{c}^p_d + \boldsymbol{t}^p_d\right) \leq \beta \overline{\boldsymbol{\theta}}^p_d + \boldsymbol{\Lambda} \boldsymbol{B} \boldsymbol{\tau} + \boldsymbol{s}^a_d \tag{4d}$$

$$0 \leq \boldsymbol{p}^a_{b,d} \leq \overline{\boldsymbol{e}}_{b,d}, \quad \forall b \in \mathcal{B} \tag{4e}$$

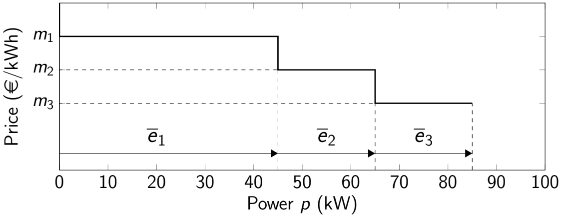
$$\boldsymbol{s}^a_d \geq 0, \tag{4f}$$

$\boldsymbol{m}_{b,d}$: Step-wise marginal utilities, $\boldsymbol{p}^a_d = \sum_{b \in \mathcal{B}} \boldsymbol{p}^a_{b,d}$

Forecasting model

$\boldsymbol{m}_{b,d}$: Step-wise marginal utilities, $\boldsymbol{p}^a_d = \sum_{b \in \mathcal{B}} \boldsymbol{p}^a_{b,d}$

Example: Three step-wise marginal utility function:



$\boldsymbol{m}_{b,d} = \boldsymbol{\nu}_b + \mathbf{Z}_d \boldsymbol{\rho}$, $\mathbf{Z}_d$: Covariates, regressors or features

IO: Inference of β, $\boldsymbol{\tau}$, and $\boldsymbol{m}_{b,d}$

Upper level (5a)–(5c)
(Minimize MAE of aggregate power)

$\downarrow$ $\boldsymbol{m}_{b,d}, \beta, \boldsymbol{\tau}$ $\qquad$ $\uparrow$ $\boldsymbol{p}^a_{b,d}$

Lower levels (5d) **for each day** d
[**Forecasting model** (4)]

$$\min_{\Xi} \sum_{d \in \mathcal{D}} \left\| \sum_{b \in \mathcal{B}} \boldsymbol{p}^a_{b,d} - \boldsymbol{p}^{a'}_d \right\|_1 \tag{5a}$$

subject to:

$$\boldsymbol{m}_{b,d} = \boldsymbol{\nu}_b + \boldsymbol{Z}_d \boldsymbol{\rho}, \quad \forall b \in \mathcal{B}, d \in \mathcal{D} \tag{5b}$$

$$\boldsymbol{\nu}_b \geqslant \boldsymbol{\nu}_{b+1}, \quad \forall b < n_B \tag{5c}$$

$$\text{Forecast. model}(\boldsymbol{m}_{b,d}, \beta, \boldsymbol{\tau}), \quad d \in \mathcal{D}. \tag{5d}$$

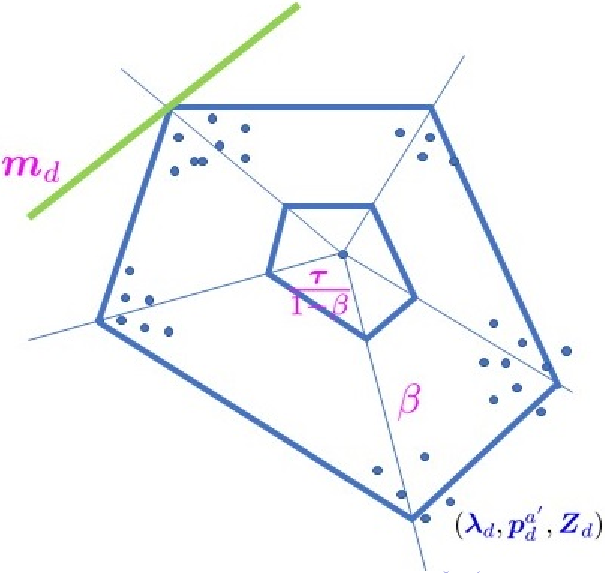
$$\Xi = \{\boldsymbol{m}_{b,d}, \boldsymbol{p}^a_{b,d}, \boldsymbol{s}^a_d, \beta, \boldsymbol{\tau}, \boldsymbol{\nu}_b, \boldsymbol{\rho}\}$$

$\boldsymbol{p}^{a'}_d$: Vector of observed aggregate power in day d

Solution approach: Regularization + Initialization[1]

[1] R. Fernández-Blanco, J.M. Morales and S. Pineda, “Forecasting the price-response of a pool of buildings via homothetic inverse optimization.” arXiv:2004.09819v2.

IO: Inference of β, $\boldsymbol{\tau}$, and $\boldsymbol{m}_{b,d}$



Data generation

We assume that the consumption of each building i for each day d is given by:

$$\min_{p_h, s_h, \theta_h} \quad \sum_{h \in \mathcal{H}} \left(p_h \lambda_h + \varrho s_h \right) \tag{6a}$$

$$\theta_h = a_1 \theta_{h-1} + (1 - a_1) \left[\theta_h^{amb} - a_2 p_h \right], \forall h \in \mathcal{H} \tag{6b}$$

$$- s_h + \underline{\theta}_h \leq \theta_h \leq \overline{\theta}_h + s_h, \quad \forall h \in \mathcal{H} \tag{6c}$$

$$0 \leq p_h \leq \overline{p}_h, \quad \forall h \in \mathcal{H} \tag{6d}$$

$$s_h \geq 0, \quad \forall h \in \mathcal{H} \tag{6e}$$

The pool demand is driven by the heterogeneity factor $\hbar$. For instance:

$$C^i \rightsquigarrow U\left[\left(1 - \hbar\right) C^p, \left(1 + \hbar\right) C^p \right]$$

with C^p being the thermal capacitance of the prototype building.

Data generation

Statistics on the aggregate power

	$\hbar = 0.1$	$\hbar = 0.75$
Maximum (kW)	541.0	218.4
Mean (kW)	64.0	42.0
# hours without consumption (%)	61.8	0.0

- Simulations are run for 100 buildings and 1872 hours (78 days) using model (6), for $\hbar = 0.1$ (low heterogeneity) and $\hbar = 0.75$ (high heterogeneity).
- Training, validation and test: 35, 35 and 7 days, respectively.
- Covariates ($\boldsymbol{Z}_d$): Ambient temperature at hours $h-2$, $h-1$, h, $h+1$ and $h+2$.
- CONOPT under Pyomo 3.7.3 for solving the regularized nonlinear programs; CPLEX for the linear programs.

Comparison Methodologies

Acronym	Methodology
hio	**The proposed homothetic IO approach**
ntd	The two-step IO approach proposed in Saez-Gallego and Morales (2018)[2]. It disregards building thermal dynamics
arimax	AutoRegressive Integrated Moving Average Model with eXogenous variables (Python)
naive	Forecast in day d = observed values in $d-1$

[2] J. Saez-Gallego and J.M. Morales (2018). Short-term forecasting of price-responsive loads using inverse optimization. *IEEE Trans. Power Systems* 9(5):4805-4814

Results

Error Metric (MAE) – Comparison of Models

Model	Low heterogeneity		High heterogeneity	
	$n_B = 1$	$n_B = 6$	$n_B = 1$	$n_B = 6$
hio	52.7	52.5	22.5	16.9
ntd	87.5	88.9	24.0	26.0
arimax	108.3	108.3	23.0	23.0
naive	90.4	90.4	24.2	24.2

To take away: The aggregate demand of a highly heterogeneous pool of buildings is much easier to predict.

Results

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To take away: *hio* systematically outperforms the rest of the methods (*arimax* is competitive in the highly heterogeneous case only).

Results

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To take away: Accounting for buildings' thermal dynamics is significantly advantageous (compare *hio* with *ntd*).

Results

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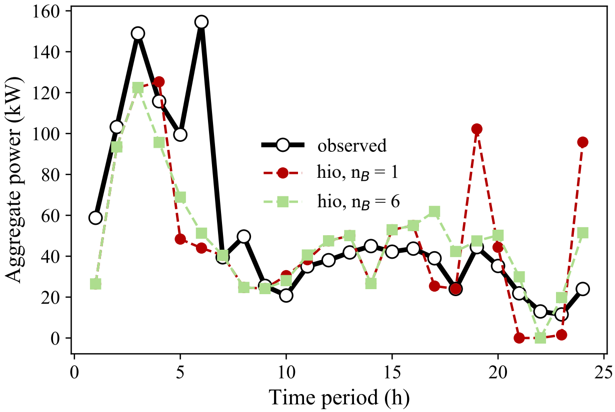
To take away: The number of marginal utility blocks has a substantial impact in the highly heterogeneous case.

Results



High heterogeneity, $n_B = 1$ vs. $n_B = 6$

To wrap up...

- A novel day-ahead forecasting technique for the aggregate demand of a pool of buidlings/TCLs
  - Homothetic transformation
  - Inverse optimization
  - Bilevel programming
- Unlike other techniques
  - It accounts for building thermal dynamics. Drastic reduction of model parameters thanks to homothety
  - The forecasting model stands for a “representative” building operational model thanks to inverse optimization
- It outperforms other techniques available in the technical literature
- Adjectives that best describe our model
  - Versatile
  - Interpretable

Contacts

Any questions?

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Further details: R. Fernández-Blanco, J.M. Morales and S. Pineda, "Forecasting the price-response of a pool of buildings via homothetic inverse optimization." Preprint available in arXiv:2004.09819v2.