A bilevel framework for decision-making under uncertainty with contextual information

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Presentation outline

- Cooking a data-driven decision under uncertainty
- Taming the wild bilevel decision framework
- Case Study: Energy producer *à la Cournot*
- Some numerical results
- Conclusions

Cooking a data-driven decision (I)

$$\min_{z \in Z} \mathbb{E}[f_0(z; y) | X = x]$$

x contextual info

y uncertain parameter

z decision

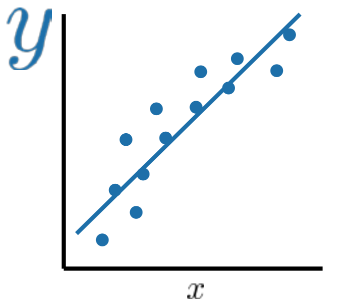
A **P**redict **O**ptimize (**PO**) recipe…

1. Select an uncertain parameter model
2. Train the model with a data set
3. Issue a point forecast for the unseen period $\tilde{t}$
4. Solve the deterministic optimization problem

$$\hat{y} = g(x; w)$$
$$w^* = \phi(g(x_t; w), y_t)$$
$$\hat{y}_{\tilde{t}} = g(x_{\tilde{t}}; w^*)$$
$$z_{\tilde{t}} = \arg\min_{z \in Z} f_0(z; \hat{y}_{\tilde{t}})$$

$g(x)$

linear case



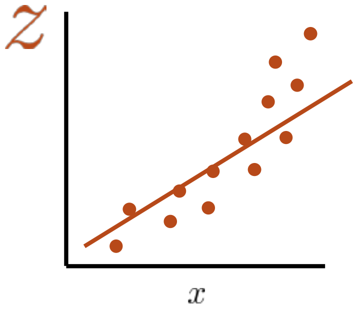
… another **D**ecision **L**earning (**DL**) recipe:

1. Select a decision model
2. Train the model with a data set
3. Compute the decision for the unseen period $\tilde{t}$

$$\hat{z} = g'(x; w')$$
$$w'^* = \phi_{f_0}(g'(x_t; w'), y_t)$$
$$\hat{z}'_{\tilde{t}} = g'(x_{\tilde{t}}; w'^*)$$

$g'(x)$

linear case



$g(x), g'(x)$ are not necessarily a linear regression of $\{(x_t, y_t)\}, \{(x_t, z_t)\}$

Cooking a data-driven decision (II)

Predict Optimize (PO) recipe

Predictive approach (FO)

$$w^{\mathrm{FO}} = \arg\min_{w\in\mathbb{R}^q} \sum_{t\in\mathcal{T}} l^{\mathrm{FO}}(\underbrace{g^{\mathrm{FO}}(x_t;w)}_{y_t^{\mathrm{FO}}}, y_t)$$

$$z_{\tilde{t}}^{\mathrm{FO}} = \arg\min_{z\in Z} f_0(z; g^{\mathrm{FO}}(x_{\tilde{t}}; w^{\mathrm{FO}}))$$

Bilevel prescriptive approach (BL)

$$w^{\mathrm{BL}} = \arg\min_{w\in\mathbb{R}^q} \sum_{t\in\mathcal{T}} f_0(\hat{z}_t; y_t)$$

$$\text{s.t.} \quad \hat{z}_t = \arg\min_{z\in Z} f_0(z; \overbrace{g^{\mathrm{BL}}(x_t;w)}^{y_t^{\mathrm{BL}}}) \quad \forall t\in\mathcal{T}$$

$$z_{\tilde{t}}^{\mathrm{BL}} = \arg\min_{z\in Z} f_0(z; g^{\mathrm{BL}}(x_{\tilde{t}}; w^{\mathrm{BL}}))$$

Decision Learning (DL) recipe

Decision Rule approach (DR[1])

$$w^{\mathrm{DR}} = \arg\min_{w\in\mathbb{R}^q} \sum_{t\in\mathcal{T}} f_0(g^{\mathrm{DR}}(x_t;w); y_t)$$

$$\text{s.t.} \quad g^{\mathrm{DR}}(x_t;w)\in Z \quad \forall t\in\mathcal{T}$$

$$z_{\tilde{t}}^{\mathrm{DR}} = g^{\mathrm{DR}}(x_{\tilde{t}}; w^{\mathrm{DR}})$$

Benchmark method (BN)

$$z_{\tilde{t}}^{\mathrm{BN}} = \arg\min_{z} f_0(z; y_{\tilde{t}})$$

Benchamark
with perfect info. of $y_{\tilde{t}}$

[1] BAN, Gah-Yi; RUDIN, Cynthia. The big data newsvendor: Practical insights from machine learning. *Operations Research*, 2019, vol. 67, no 1, p. 90-108.

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$$w^{\mathrm{DR}} = \arg\min_{w \in \mathbb{R}^q} \sum_{t \in \mathcal{T}} f_0(g^{\mathrm{DR}}(x_t; w); y_t)$$

$$\text{s.t.} \quad g^{\mathrm{DR}}(x_t; w) \in Z \quad \forall t \in \mathcal{T}$$

$$z_{\tilde{t}}^{\mathrm{DR}} = g^{\mathrm{DR}}(x_{\tilde{t}}; w^{\mathrm{DR}})$$

- ✓ Perform well in most tasks
- ✓ Simpler
- × Suboptimal for decision-making

- ✓ Yields better decisions
- × More complex
- × Issues for linear f_0

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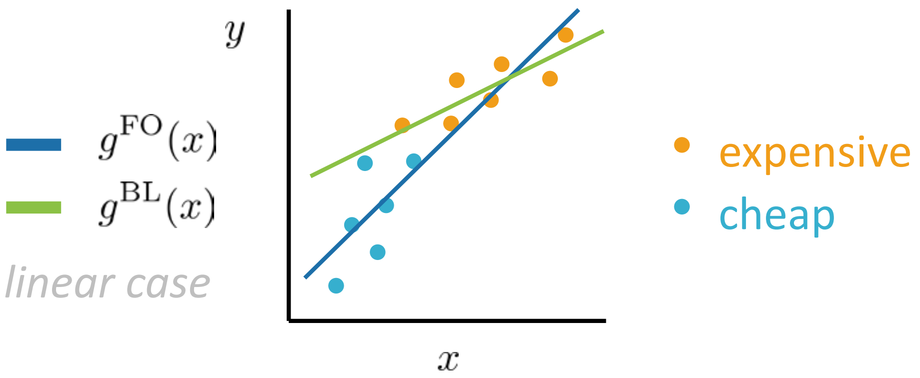

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Taming the wild bilevel (I)

$$w^{\mathrm{BL}} = \arg\min_{w \in \mathbb{R}^q} \sum_{t \in \mathcal{T}} f_0(\hat{z}_t; y_t)$$

Lower Level

$$\left.\begin{aligned} \text{s.t. } \; & \hat{z}_t = \arg\min_z f_0(z; g^{\mathrm{BL}}(x_t; w)) \;\; \forall t \in \mathcal{T} \\ & \left.\begin{aligned} f_i(z_t) \le 0, \;\; \forall i \\ h_j(z_t) = 0, \;\; \forall j \end{aligned}\right\} \; \forall t \in \mathcal{T} \end{aligned}\right\} \text{Feasible set } Z$$

KKT

optimality conditions

$$w^{\mathrm{BL}} = \arg\min_{w, \hat{z}_t, \lambda_{it}, \upsilon_{it}} \sum_{t \in \mathcal{T}} f_0(\hat{z}_t; y_t)$$

$$\text{s.t. } \nabla f_0(\hat{z}_t, g^{\mathrm{BL}}(x_t, w)) + \sum_{i=1}^{I} \lambda_{it} \nabla f_i(\hat{z}_t) + \sum_{j=1}^{J} \upsilon_{jt} \nabla h_j(\hat{z}_t) = 0, \;\; \forall t \in \mathcal{T} \qquad \text{(stationarity)}$$

$$\left.\begin{aligned} & f_i(\hat{z}_t) \le 0, \;\; \forall i \\ & h_j(\hat{z}_t) = 0, \;\; \forall j \\ & \lambda_{it} \ge 0, \;\; \forall i \\ & \lambda_{it} f_i(\hat{z}_t) = 0, \;\; \forall i \end{aligned}\right\} \; \forall t \in \mathcal{T}$$

(primal feasibility)

(dual feasibility)

(complementary slackness)

Taming the wild bilevel (II)

$$\lambda_{it} f_i(\hat{z}_t) = 0, \quad \forall i, \quad \forall t \in \mathcal{T}$$

(complementary slackness)

Approach 1: **Regularization**

$$-\sum_{\forall it} \lambda_{it} f_i(\hat{z}_t) \leq \epsilon$$

$$\epsilon \geq 0, \quad \epsilon \to 0$$

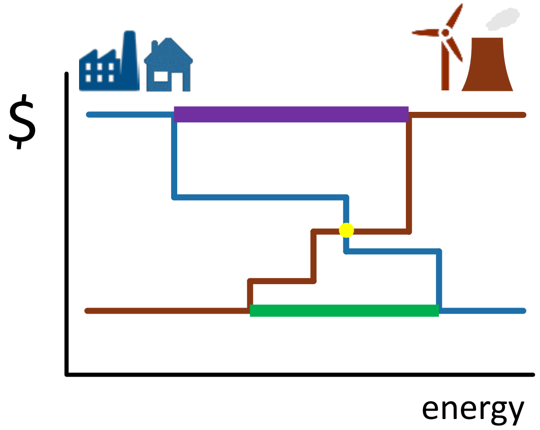
Approach 2: **Big-M**

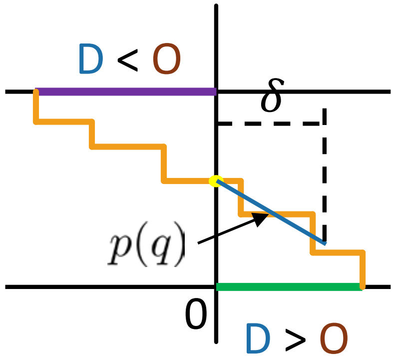
$$\left.\begin{array}{l} \lambda_{it} \leq u_{it} M^D, \\ f_i(\hat{z}_t) \geq (u_{it} - 1) M^P, \\ u_{it} \in \{0, 1\}, \end{array}\right\} \forall i, \forall t \in \mathcal{T}$$

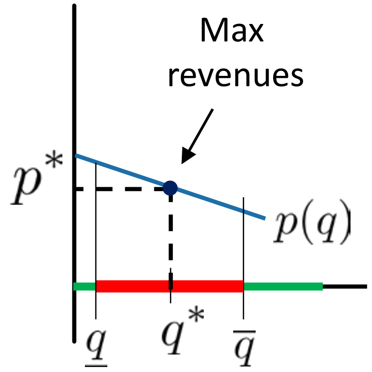
Case study *à la Cournot* (I)

$$q^* = \arg \max_{\underline{q} \le q \le \overline{q}} \; p(q)q - c(q) \qquad \begin{cases} p(q) & = \alpha - \beta q \\ c(q) & = c_2 q^2 + c_1 q + c_0 \end{cases}$$

$$q^* = \arg \min_{\underline{q} \le q \le \overline{q}} \; \beta' q^2 - \alpha' q \qquad \begin{cases} \beta' = \beta + c_2 \\ \alpha' = \alpha - c_1 \end{cases}$$







Some assumptions:

- Our behaviour does not influence the rest of the competitors.
- Our offer is just a single quantity.
- The transmission grid is neglected.
- A linear function can approximate the residual demand.

Case study *à la Cournot* (II)

Approach BN

$$q_t^{BN} = \arg \min_{\underline{q} \le q_t \le \overline{q}} \; \beta_t' q_t^2 - \alpha_t' q_t \qquad \begin{cases} q_t^{BN} & = \arg\min_{\underline{q} \le q_t \le \overline{q}} \; q_t^2 - \gamma_t q_t \\ \gamma_t & = \alpha_t' / \beta_t', \;\; \beta_t' > 0 \end{cases} \qquad g(x; w) = w^T x$$

Approach FO

$$w^{\text{FO}} = \arg \min_{w \in \mathbb{R}^q} \sum_{t \in \mathcal{T}} (\gamma_t - \underbrace{w^T x_t}_{\gamma_t^{\text{FO}}})^2$$

$$q_{\tilde{t}}^{\text{FO}} = \arg \min_{\underline{q} \le q \le \overline{q}} \; q^2 - \underbrace{(w^{\text{FO}})^T x_{\tilde{t}}}_{\gamma_{\tilde{t}}^{\text{FO}}} \; q$$

Approach BL

$$\begin{aligned} w^{\text{BL}} = \arg \min_{w \in \mathbb{R}^q} & \sum_{t \in \mathcal{T}} \beta_t' \hat{q}_t^2 - \alpha_t' \hat{q}_t \\ \text{s.t.} \;\; & \hat{q}_t = \arg \min_{\underline{q} \le q \le \overline{q}} q^2 - \underbrace{w^T x_t}_{\gamma_t^{\text{BL}}} q, \;\; \forall t \in \mathcal{T} \end{aligned}$$

$$q_{\tilde{t}}^{\text{BL}} = \arg \min_{\underline{q} \le q \le \overline{q}} \; q^2 - \underbrace{(w^{\text{BL}})^T x_{\tilde{t}}}_{\gamma_{\tilde{t}}^{\text{BL}}} \; q$$

Approach DR

$$\begin{aligned} w^{\text{DR}} = \arg \min_{w \in \mathbb{R}^q} & \sum_{t \in \mathcal{T}} \beta_t' (w^T x_t)^2 - \alpha_t' w^T x_t \\ \text{s.t.} \;\; & \underline{q} \le \underbrace{w^T x_t}_{q_t^{\text{DR}}} \le \overline{q}, \;\; \forall t \in \mathcal{T} \end{aligned}$$

$$q_{\tilde{t}}^{\text{DR}} = (w^{\text{DR}})^T x_{\tilde{t}}$$

Case study *à la Cournot* (III)

$$w^{\mathrm{BL}} = \arg\min_{w\in\mathbb{R}^q} \sum_{t\in\mathcal{T}} \beta_t'\hat{q}_t^2 - \alpha_t'\hat{q}_t$$
$$\text{s.t. } \hat{q}_t = \arg\min_{\underline{q}\le q_t\le\overline{q}} q_t^2 - w^T x_t q_t, \quad \forall t\in\mathcal{T}$$

$$w^{\mathrm{BL}} = \arg\min_{w,\hat{q}_t,\lambda_{1t},\lambda_{2t}} \sum_{t\in\mathcal{T}} \beta_t'\hat{q}_t^2 - \alpha_t'\hat{q}_t$$
$$\text{s.t. } 2\hat{q}_t - w^T x_t - \lambda_{1t} + \lambda_{2t} = 0, \quad \forall t\in\mathcal{T}$$
$$\left.\begin{array}{l} \underline{q} \le \hat{q}_t \le \overline{q} \\ \lambda_{1t}, \lambda_{2t} \ge 0 \end{array}\right\} \forall t\in\mathcal{T}$$

BL-R approach

NLP

CONOPT

$$\sum_{t\in\mathcal{T}} \lambda_{1t}(\hat{q}_t - \underline{q}) + \lambda_{2t}(\overline{q} - \hat{q}_t) \le \epsilon$$

Iterative solved for $\epsilon \to 0$

↑ fast, ↓ non-optimal

BL-M approach

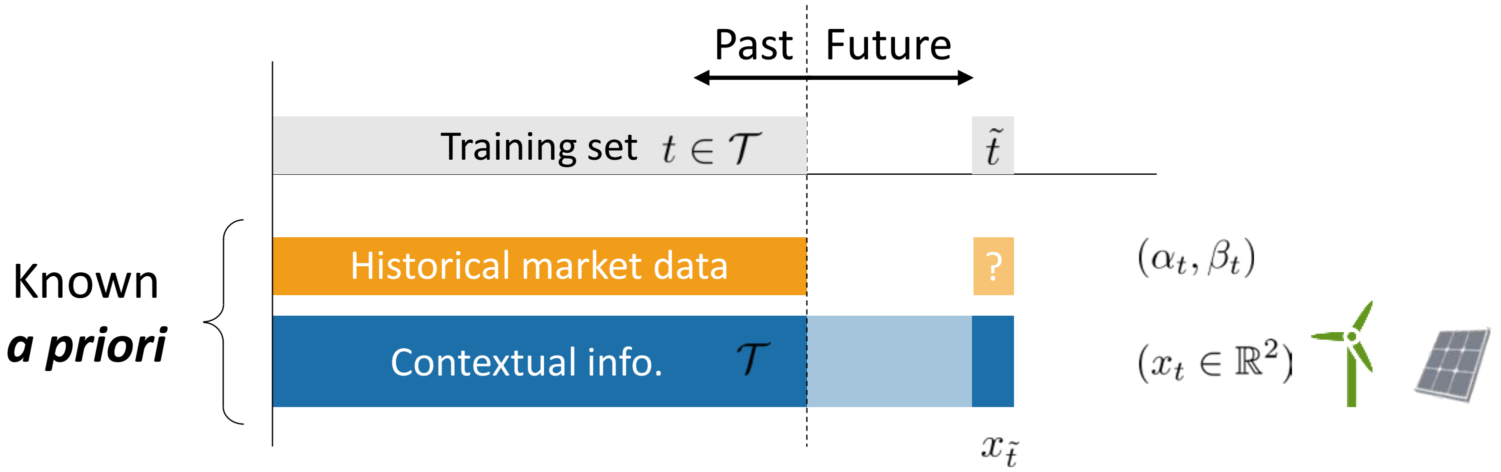
$$\left.\begin{array}{l} \lambda_{kt} \le u_{kt}M^D, \quad k=1,2 \\ \hat{q}_t - \underline{q} \le (1-u_{1t})M^P \\ \overline{q} - \hat{q}_t \le (1-u_{2t})M^P \\ u_{1t}, u_{2t} \in \{0,1\} \end{array}\right\} \forall t\in\mathcal{T}$$

MIQP

CPLEX

↑ optimality, ↓ slow, big-M tunning

Some numerical results (I)



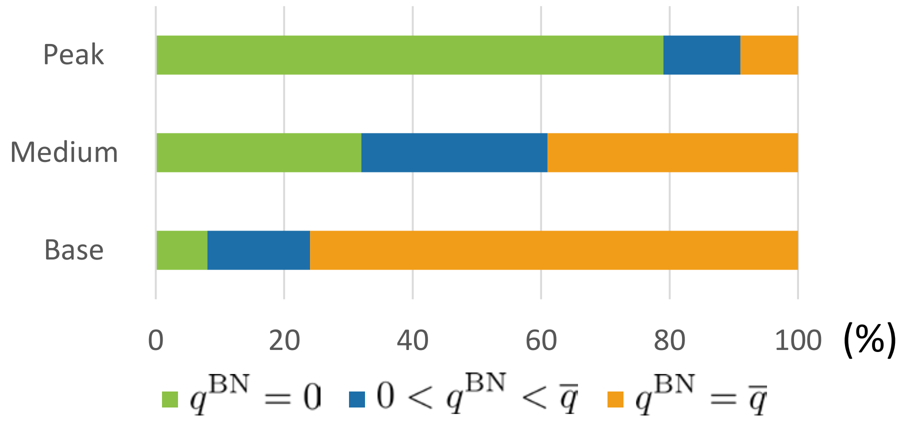
Experimental setup

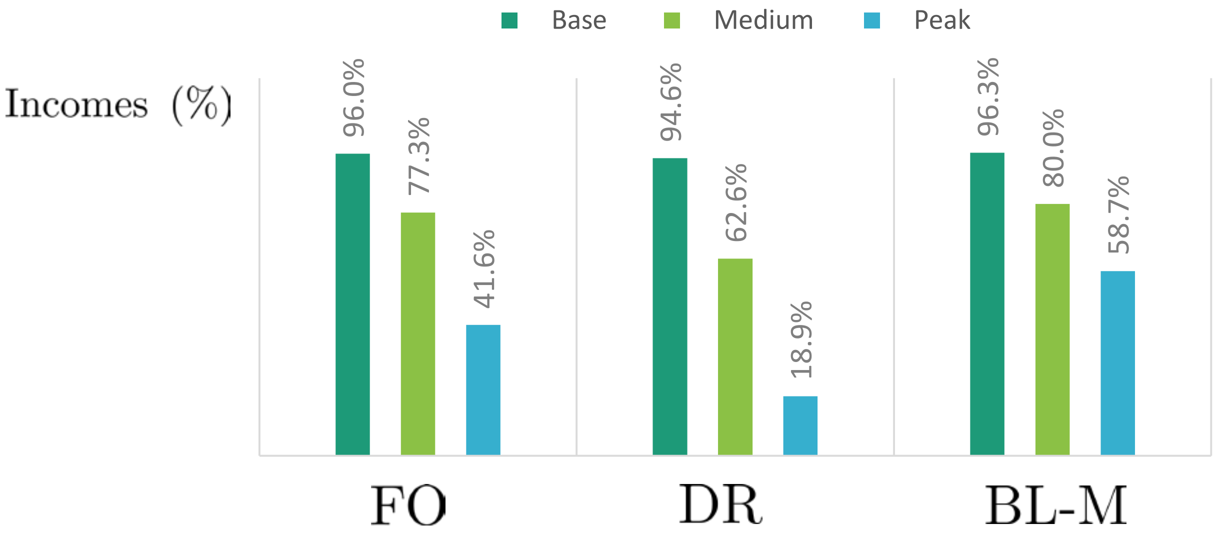
- Data set of 8600 hours (data points).
- Bins of 200 points, 80% training, 20% test.
- Reshuffle training test assignment 5 times.
- Contextual info: Wind and Solar power production forecast $(x_{\tilde{t}} \in \mathbb{R}^2)$

Some numerical results (II)

	c_1 (€/MWh)	$\overline{q}$ (MW)
Base	10	1000
Medium	35	500
Peak	50	250

$c_2 = 0.005$ €/MWh^2





	Infeasibility (DR)
Base	4.9%
Medium	1.7%
Peak	0.1%

Conclusions

- Novel data-driven framework for conditional stochastic optimization, where parameters are formulated as a function of some covariates.

- Application to the problem of a Cournot producer supplying to the residual demand.

- Numerical experiments show our proposal can significantly increase the competitive edge of the Cournot producer.

THANKS!

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Checkout more at:

M. A. Muñoz, S. Pineda, J. M. Morales, **A bilevel framework for decision-making under uncertainty with contextual information**. *arXiv preprint arXiv:2008.01500*, 2020.