Electricity market equilibrium models in a two-market setup

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CMS 2016 Salamanca, Spain
Joint work with Salvador Pineda and Trine Krogh Boomsma

June 1st 2016

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1. Two-market setup
2. Open- and closed-loop formulation
3. Results
4. Contributions and further work

Two-market setup

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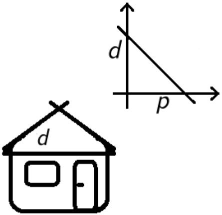

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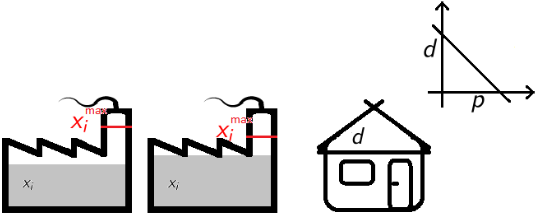

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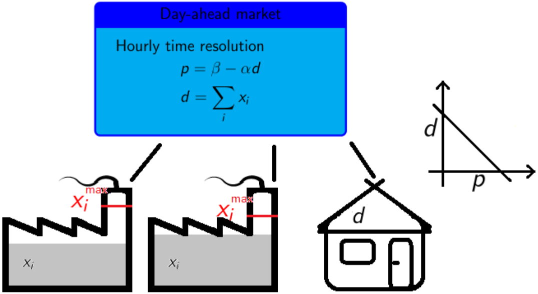

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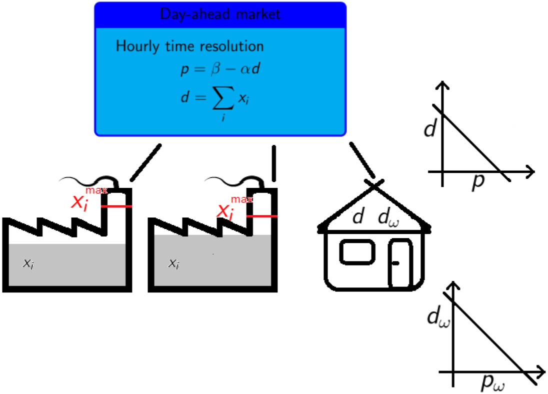


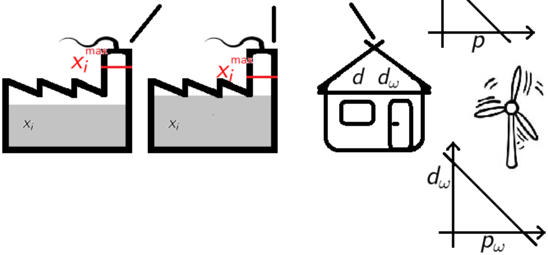
Two-market setup

Day-ahead market

Hourly time resolution

$$p = \beta - \alpha d$$

$$d = \sum_i x_i$$



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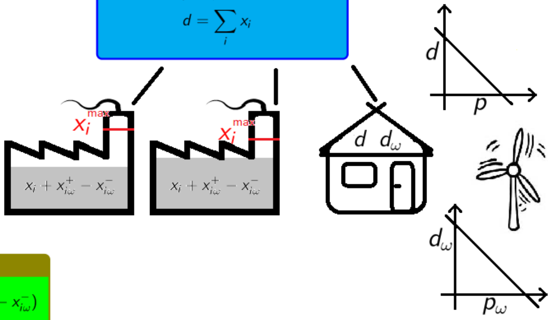

Unit i

$$\max \; px_i + \sum_{t \in T_h} \mathbb{E}\Big[p_\omega(x_{i\omega}^+ - x_{i\omega}^-) - \mathrm{cost}(x_i + x_{i\omega}^+ - x_{i\omega}^-)\Big]$$

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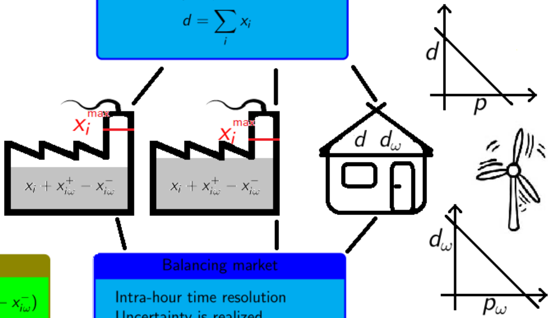

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Balancing market

Intra-hour time resolution
Uncertainty is realized

$$p_\omega = \beta_\omega - \alpha d_\omega$$
$$d_\omega = \sum_i (x_i + x_{i\omega}^+ - x_{i\omega}^-)$$

Two-market setup

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$$p = \beta - \alpha d$$
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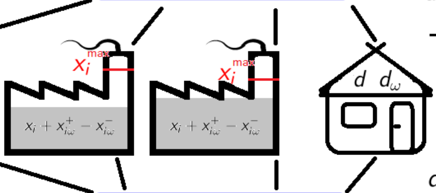
Arbitrageur

Max profits
Buy/sell in DA
Sell/buy in BAL
$p = \mathbb{E}[p_\omega]$



Unit i

$$\max \ p x_i + \sum_{t \in T_h} \mathbb{E}\Big[p_\omega(x^+_{i\omega} - x^-_{i\omega}) - \text{cost}(x_i + x^+_{i\omega} - x^-_{i\omega})\Big]$$

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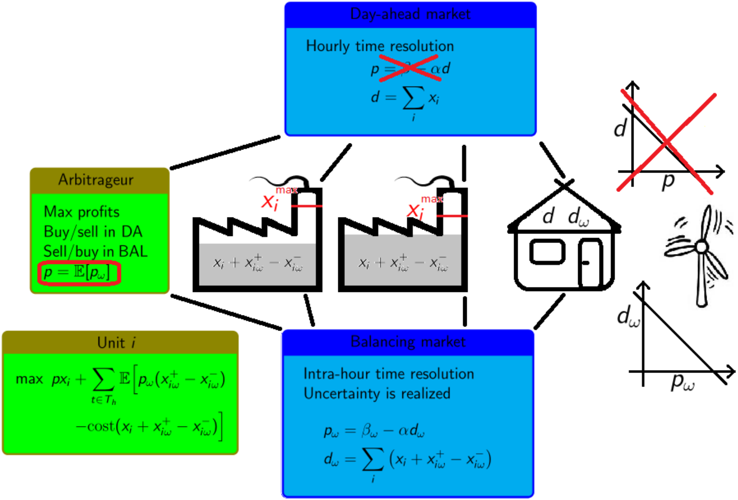

Formulations

- Open-loop
- Closed-loop (subgame perfect equilibrium)
- Supply function equilibrium and variations

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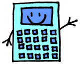
- Open-loop 
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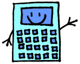
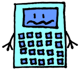
- Open-loop 
- Closed-loop (subgame perfect equilibrium) 
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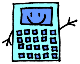
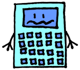
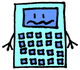
- Open-loop 
- Closed-loop (subgame perfect equilibrium) 
- Supply function equilibrium and variations 

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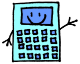
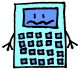
- Open-loop 
- Closed-loop (subgame perfect equilibrium) 
- When is open-loop solution $\neq$ closed-loop solution?

Open-loop

$$\max_{x_i, x_{i\omega}^+, x_{i\omega}^-} \; p x_i + \mathbb{E}[p_\omega (x_{i\omega}^+ - x_{i\omega}^-)$$
$$- cost(x_i + x_{i\omega}^+ - x_{i\omega}^-)]$$
$$\text{S.t. } 0 \leq x_i \leq x_i^{\max}$$
$$0 \leq x_{i\omega}^+ \leq x_i - x_i^{\max}$$
$$0 \leq x_{i\omega}^- \leq x_i$$

Closed-loop stage 1

$$\max_{x_i, x_{i\omega}^+, x_{i\omega}^-} \quad px_i + \mathbb{E}[p_\omega(x_{i\omega}^+ - x_{i\omega}^-) - cost(x_i + x_{i\omega}^+ - x_{i\omega}^-)]$$

S.t. $0 \leq x_i \leq x_i^{\max}$

Stage 2 KKT

Open-loop

$$\max_{x_i, x_{i\omega}^+, x_{i\omega}^-} \quad px_i + \mathbb{E}[p_\omega(x_{i\omega}^+ - x_{i\omega}^-) - cost(x_i + x_{i\omega}^+ - x_{i\omega}^-)]$$

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$$\text{S.t. } 0 \leq x_i \leq x_i^{\max}$$

$$0 \leq x_{i\omega}^+ \leq x_i - x_i^{\max}$$

$$0 \leq x_{i\omega}^- \leq x_i$$

Closed-loop stage 2

$$\max_{x_{i\omega}^+, x_{i\omega}^-} \; p_\omega (x_{i\omega}^+ - x_{i\omega}^-) - cost(x_i + x_{i\omega}^+ - x_{i\omega}^-)$$

$$\text{S.t. } 0 \leq x_{i\omega}^+ \leq x_i - x_i^{\max}$$

$$0 \leq x_{i\omega}^- \leq x_i$$

Closed-loop stage 1

$$\max_{x_i, x_{i\omega}^+, x_{i\omega}^-} \quad p x_i + \mathbb{E}[p_\omega (x_{i\omega}^+ - x_{i\omega}^-)$$
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$$\text{Stage 2 KKT}$$

Open-loop

$$\max_{x_i, x_{i\omega}^+, x_{i\omega}^-} \quad p x_i + \mathbb{E}[p_\omega (x_{i\omega}^+ - x_{i\omega}^-)$$
$$- \mathit{cost}(x_i + x_{i\omega}^+ - x_{i\omega}^-)]$$
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Closed-loop stage 2

$$\max_{x_{i\omega}^+, x_{i\omega}^-} \quad p_\omega (x_{i\omega}^+ - x_{i\omega}^-)$$
$$- \mathit{cost}(x_i + x_{i\omega}^+ - x_{i\omega}^-)$$
$$\text{S.t. } 0 \leq x_{i\omega}^+ \leq x_i - x_i^{\max}$$
$$0 \leq x_{i\omega}^- \leq x_i$$

Market clearing

$$p = \mathbb{E}[p_\omega]$$
$$p_\omega = \beta_\omega - \alpha \sum_i (x_i + x_{i\omega}^+ - x_{i\omega}^-)$$

Results

- 3 identical power production units, $x^{\max} = 60$MWh

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- $p_\omega = 100 - \sum_i (x_i + x_i^+ - x_i^-)$.

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Open-loop

$x_i{}^*$	x_{it}^{+*}	x_{it}^{-*}	Total prod	Price DA	Price bal	Cost	Profit
12.50	0.00	0.00	12.50	62.50	62.50	625.00	156.25

Closed-loop

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- $p_{1\omega} = 60 - \sum_i (x_i/2 + x_i^+ - x_i^-)$.

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- $p_{1\omega} = 60 - \sum_i (x_i/2 + x_i^+ - x_i^-)$.
- $p_{2\omega} = 140 - \sum_i (x_i/2 + x_i^+ - x_i^-)$.

Results

- One time period in DA and two in BAL
- $p_{1\omega} = 60 - \sum_i (x_i/2 + x_i^+ - x_i^-)$.
- $p_{2\omega} = 140 - \sum_i (x_i/2 + x_i^+ - x_i^-)$.

Open-loop

t	${x_i}^*$	${x_{it}^+}^*$	${x_{it}^-}^*$	Total prod	Price DA	Price bal	Cost	Profit
1	42.50	0.00	17.50	3.75	62.50	48.75	275.00	465.62
2		0.00	0.00	21.25		76.25	1062.50	

Closed-loop

t	${x_i}^*$	${x_{it}^+}^*$	${x_{it}^-}^*$	Total prod	Price DA	Price bal	Cost	Profit
1	41.00	0.00	11.62	8.87	52.75	33.38	501.87	252.41
2		2.13	0.00	22.62		72.13	1173.75	

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- Market power analysis when access to balancing market is limited
- Including ramp rates and network

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Further work includes

- Look into the inverse demand functions

- Market power analysis when access to balancing market is limited
- Including ramp rates and network
- Include stochastic producers
- Efficient solutions methods e.g. parallelization.

References

- Wogrin et al. (2013) *Open versus closed loop capacity equilibria in electricity markets under perfect and oligopolistic competition*
- Shanbhag et al. (2011) *A Complementarity Framework for Forward Contracting Under Uncertainty*
- Allaz (1992) *Oligopoly, uncertainty and strategic forward transactions*