On the Inefficiency of the Merit Order in Forward Electricity Markets with Uncertain Supply

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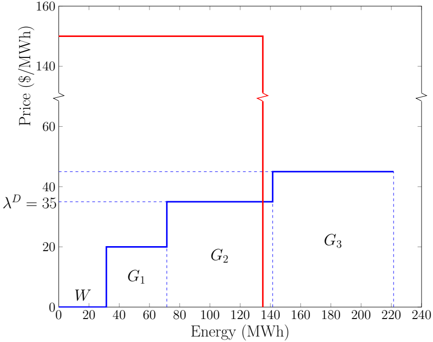
Uncertain supply

Mainly power supply driven by short-term weather conditions

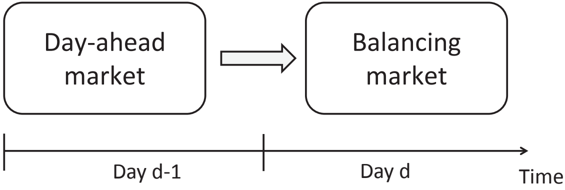
Electricity markets with uncertain supply are the future as we transition to a low-carbon energy society

Merit Order



Rule to clear the electricity market whereby generators with the lowest marginal costs are dispatched first

Our market framework



- A day-ahead market to plan the operation of the power system: it captures the cost of the *predictable/explainable* component of the uncertain supply
- A real-time market to ensure the continuous balance of generation and demand: it uncovers the cost of the *unpredictable* part of the uncertain supply

Our aim

Our aim is to understand why and in which cases the traditional merit-order dispatch becomes inefficient under uncertain supply, by using the stochastic two-stage market as an ideal benchmark

Conventional or inefficient two-stage market

Day-ahead market

$$\underset{\mathbf{p},\boldsymbol{\delta}^f}{\text{Minimize}} \quad \mathcal{C}^f(\mathbf{p}) \tag{1a}$$

$$\text{s.t.} \quad \mathbf{h}^{\mathrm{f}}(\mathbf{p},\boldsymbol{\delta}^f;\boldsymbol{l}) = \mathbf{0} : \boldsymbol{\lambda}^f, \tag{1b}$$

$$\mathbf{g}^{\mathrm{f}}(\mathbf{p},\boldsymbol{\delta}^f;\widehat{\boldsymbol{W}}) \leqslant \mathbf{0} : \boldsymbol{\mu}^f, \tag{1c}$$

Balancing market $(\boldsymbol{W}_\omega, \omega \in \Omega)$

$$\underset{\mathbf{r}_\omega,\boldsymbol{\delta}^b_\omega}{\text{Minimize}} \quad \mathcal{C}^b(\mathbf{r}_\omega) \tag{2a}$$

$$\text{s.t.} \quad \mathbf{h}^{\mathrm{b}}(\mathbf{r}_\omega,\boldsymbol{\delta}^b_\omega;\boldsymbol{\delta}^{f\star},\mathbf{p}^\star,\boldsymbol{W}_\omega) = \mathbf{0} : \boldsymbol{\lambda}^b_\omega, \tag{2b}$$

$$\mathbf{g}^{\mathrm{b}}(\mathbf{r}_\omega,\boldsymbol{\delta}^b_\omega;\mathbf{p}^\star,\boldsymbol{W}_\omega) \leqslant \mathbf{0}, \tag{2c}$$

- Stochastic power production $\boldsymbol{W}$ defined on some probability space $(\Omega, \mathcal{F}, P)$. $\widehat{\boldsymbol{W}}$ is its expectation and $\boldsymbol{W}_\omega, \omega \in \Omega$, a particular realization
- $\mathbf{p}$ and $\mathbf{r}_\omega$ stands for the forward generation dispatch and the balancing re-dispatch, respectively. $\boldsymbol{\delta}^f$ and $\boldsymbol{\delta}^b_\omega$ are the vectors of network-state variables, and $\boldsymbol{l}$ is the vector of nodal demands
- No intra-area market coordination: the day-ahead and the balancing markets are sequentially and independently cleared

Stochastic or efficient two-stage market

Day-ahead market

$$\underset{\mathbf{p},\boldsymbol{\delta}^f,\mathbf{r}(\omega),\delta^b(\omega)}{\text{Minimize}} \quad \mathcal{C}^f(\mathbf{p}) + E_\Omega\left[\mathcal{C}^b(\mathbf{r}(\omega))\right] \tag{3a}$$

$$\text{s.t.} \quad \mathbf{h}^{\mathrm{f}}(\mathbf{p},\boldsymbol{\delta}^f;\boldsymbol{l}) = \mathbf{0} : \boldsymbol{\nu}^f, \tag{3b}$$

$$\mathbf{g}^{\mathrm{f}}(\mathbf{p},\boldsymbol{\delta}^f) \leqslant \mathbf{0}, \tag{3c}$$

$$\mathbf{h}^{\mathrm{b}}(\mathbf{r}(\omega),\boldsymbol{\delta}^b(\omega),\boldsymbol{\delta}^f,\mathbf{p};\boldsymbol{W}(\omega)) = \mathbf{0}, \forall\omega\in\Omega \tag{3d}$$

$$\mathbf{g}^{\mathrm{b}}(\mathbf{r}(\omega),\boldsymbol{\delta}^b(\omega),\mathbf{p};\boldsymbol{W}(\omega)) \leqslant \mathbf{0}, \forall\omega\in\Omega \tag{3e}$$

Balancing market $(W_\omega, \omega\in\Omega)$

$$\underset{\mathbf{r}_\omega,\boldsymbol{\delta}^b_\omega}{\text{Minimize}} \quad \mathcal{C}^b(\mathbf{r}_\omega) \tag{2a}$$

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$$\mathbf{g}^{\mathrm{b}}(\mathbf{r}_\omega,\boldsymbol{\delta}^b_\omega;\mathbf{p}^\star,\boldsymbol{W}_\omega) \leqslant \mathbf{0}, \tag{2c}$$

- $E_\Omega\left[\mathcal{C}^b(\mathbf{r}(\omega))\right] = \int_\Omega \mathcal{C}^b(\mathbf{r}(\omega))f(\omega)d\omega$, with $f(\cdot)$ the pdf of $\mathbf{W}$
- It makes use of a full probabilistic characterization of the uncertain supply $\mathbf{W}$ (typically, a discrete approximate model of $\mathbf{W}$ in the form of scenarios is considered instead)
- *Perfect* intra-area market coordination: the clearings of the day-ahead and the balancing markets are co-optimized

Stochastic or efficient two-stage market

Day-ahead market

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$$\mathbf{g}^{\mathrm{b}}(\mathbf{r}_\omega,\boldsymbol{\delta}^b_\omega;\mathbf{p}^\star,\boldsymbol{W}_\omega) \leqslant \mathbf{0}, \tag{2c}$$

It delivers the highest achievable social welfare in the long run. However:

- Forward prices $\boldsymbol{\nu}^f$ do not clear the day-ahead market [Morales et al., 2012]: Flexible power producers may be dispatched in a virtual loss-making position
- Only revenue adequate in expectation, but not per scenario [Pritchard et al., 2010]
- It calls for re-centralization: market agents must agree on their perception of $\boldsymbol{W}$
- It violates the widely accepted merit order and is computationally intensive

Stochastic or efficient two-stage market

Day-ahead market

$$\underset{\mathbf{p},\boldsymbol{\delta}^f,\mathbf{r}(\omega),\delta^b(\omega)}{\text{Minimize}} \quad \mathcal{C}^f(\mathbf{p}) + E_\Omega\left[\mathcal{C}^b(\mathbf{r}(\omega))\right] \tag{3a}$$

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Alternatives to approach the efficient two-stage market:

- New market products, such as the CAISO and MISO Flexiramp markets [Wang and Hobbs, 2014, Navid and Rosenwald, 2012]
- Only the maximum dispatchable amount of stochastic power production $\boldsymbol{W}$ is centrally controlled. All other market rules remain unchanged [Morales et al., 2014]
- Capacity payments to increase the utilization of (expensive) flexible power generation (working paper by T. Rintamäki, A. Siddiqui and A. Salo)
- **Some people argue that convergence (virtual) bidding may do the trick**

Simple models for insight

We assume:

- A single-node power system (i.e., with infinite transmission capacity)
- Three types of generation technologies: inflexible (p_I), flexible (p_F) and of uncertain availability (p_W), with marginal costs equal to c_I, c_F, and 0, respectively
- The flexible and inflexible generating units have capacities $\overline{P}_F$ and $\overline{P}_I$, respectively
- c_F^+ and c_F^- are the marginal cost and utility associated with the provision of upward and downward regulation, in that order
- $c_F^+ \geqslant c_F \geqslant c_F^- \geqslant 0$ (the flexible power producer is rational)

Simple models for insight

In this presentation we will limit ourselves to the subset of cases given by:

- $\overline{P}_F \geqslant \overline{W}$ (capacity adequate power system)
- $\overline{P}_I \geqslant L$, with L being the system load
- $c_I < c_F$ (probably the most interesting subset of cases as we want to identify conditions under which the merit order is broken)

Single-node power system

Proposition (actually a corollary of a more general result...)

The marginal expected cost of operating the power system previously described, $\frac{dz}{dL}(L)$, is given by:

$$\frac{dz}{dL}(L) = min\left(c_I,\ \ c_F^{+}F(L),\ \ c_F - c_F^{-}\left(1 - F(L)\right)\right)\ , \tag{5}$$

where $F(L) = P\left(W \leqslant L\right)$ is the cdf that characterizes the stochastic power production W.

Remark: Despite the simplicity of our power system, the marginal expected cost is an intricate function of the cost structure of the flexible and inflexible power capacities and the probability distribution of the uncertain supply.

Single-node power system

Proposition cont. (actually a corollary of a more general result...)

$\frac{dz}{dL}(L)$ prompts the following dispatch rule:

If $L_2 \leqslant L_1$

$$\begin{cases} p_W = L, \quad p_I = 0, \quad p_F = 0, \ 0 \leqslant L \leqslant L_2; \\ p_W = L_2, \quad p_I = L - L_2, \quad p_F = 0, \ L > L_2. \end{cases}$$

Else

$$\begin{cases} p_W = L, \quad p_I = 0, \quad p_F = 0, \ 0 \leqslant L \leqslant L_1; \\ p_W = L_1, \quad p_I = 0, \quad p_F = L - L_1, \ L_1 < L \leqslant L_3; \\ p_W = L_1, \quad p_I = L - L_3, \quad p_F = L_3 - L_1, \ L > L_3. \end{cases}$$

where

$$L_1 = F^{-1}\left(\frac{c_F - c_F^-}{c_F^+ - c_F^-}\right); \quad L_2 = F^{-1}\left(\frac{c_I}{c_F^+}\right); \quad L_3 = F^{-1}\left(1 - \frac{c_F - c_I}{c_F^-}\right)$$

Single-node power system

Remark: If $L_2 > L_1$, the efficient two-stage market may provide a dispatch solution that breaks the merit order![1]

[1] Unless I state it otherwise, "breaking the merit order" means dispatching the expensive flexible power capacity over the (cheaper) inflexible one

Relevant cases I

Fully *inflexible* power system

Consider the stylized power system previously described, where, in addition, we have that $c_F^+ \to \infty$ and $c_F^- \to -\infty$. The marginal expected cost function of this *fully inflexible* power system is given by

$$\frac{dz}{dL}(L) = c_I \ , \tag{6}$$

which leads to the following dispatch rule:

$$p_W = 0, \quad p_I = L, \quad p_F = 0, \ \ L \geqslant 0$$

Relevant cases II

Fully *flexible* power system

Consider the stylized power system previously described, where, in addition, all the conventional generating capacity is flexible with $c_F^+ = c_F^- = c_I = c_F$. The marginal expected cost function of this *fully flexible* power system is given by

$$\frac{dz}{dL}(L) = c_F F(L) \; , \tag{7}$$

which is associated with the following dispatch rule:

Any $(p_W, p_F) : p_W + p_F = L,$ with $0 \leqslant p_W \leqslant \overline{W}$ and $\; L \geqslant 0,$ is optimal.

$\overline{W}$ is the capacity of the stochastic power production.

Relevant cases II

Fully *flexible* power system

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$\overline{W}$ is the capacity of the stochastic power production.

Remark: If the system is fully flexible, the merit-order dispatch rule provides an optimal dispatch solution

Relevant cases III

Equal forward and real-time marginal costs

Consider the stylized power system previously described. Further, assume that $c_F^+ = c_F^- = c_F$. The marginal expect cost function of this power system is given by

$$\frac{dz}{dL}(L) = min\left(c_I, c_F F(L)\right) \ , \tag{8}$$

which prompts the following dispatch rule:

If $0 \leqslant L \leqslant L_2 = F^{-1}\left(\frac{c_I}{c_F}\right)$

Any $(p_W, p_F) : p_W + p_F = L,$ with $0 \leqslant p_W \leqslant \overline{W}$ is optimal.

Else

Any $(p_W, p_F, p_I) : p_W + p_F = L_2, \ \ p_I = L - L_2$ with $0 \leqslant p_W \leqslant \overline{W}$ is optimal.

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Any $(p_W, p_F) : p_W + p_F = L,$ with $0 \leqslant p_W \leqslant \overline{W}$ is optimal.

Else

Any $(p_W, p_F, p_I) : p_W + p_F = L_2, \ \ p_I = L - L_2$ with $0 \leqslant p_W \leqslant \overline{W}$ is optimal.

Remark: If the forward and real-time marginal production costs of the flexible power capacity are equal, there exists a dispatch solution (the one with $p_F^\star = 0$) that preserves the merit order and that is optimal under the efficient two-stage market

Example: Uncertain supply

- Wind power production with a limited capacity $\overline{W}$ of 100 MW
- Wind power capacity factor follows a Beta distribution with a mean (κ) and a standard deviation (σ) that are linked together through the empirical relationship [Fabbri et al., 2005]

$$\sigma = 0.01837 + 0.20355 \cdot \kappa \tag{9}$$

- The shape parameters α and β of the Beta distribution modeling the wind power capacity factor are, consequently, computed as follows:

$$\alpha = \frac{(1-\kappa)\cdot\kappa\cdot\kappa}{\sigma^2} - \kappa, \qquad \beta = \alpha\left(\frac{1-\kappa}{\kappa}\right) \tag{10}$$

- $E_\Omega[W(\omega)] = \widehat{W} = k\overline{W}$

Example: Four cases

Case a) $c_I = \$30/\text{MWh}$, $c_F = \$35/\text{MWh}$, $c_F^+ = \$40/\text{MWh}$, $c_F^- = \$30/\text{MWh}$.

$\frac{c_I}{c_F^+} = \frac{30}{40} > \frac{c_F - c_F^-}{c_F^+ - c_F^-} = \frac{5}{10} \Rightarrow$ the merit order is broken

Case b) $c_I = \$30/\text{MWh}$, $c_F = c_F^+ = c_F^- = \$35/\text{MWh}$.

The spread between forward and real-time marginal costs is 0 $\Rightarrow$ the merit order is respected

Case c) $c_I = \$30/\text{MWh}$, $c_F = c_F^+ = \$35/\text{MWh}$, $c_F^- = \$30/\text{MWh}$.

Asymmetric balancing costs: upward regulation is less costly than downward regulation (atypical)

$\frac{c_I}{c_F^+} = \frac{30}{35} < \frac{c_F - c_F^-}{c_F^+ - c_F^-} = 1 \Rightarrow$ the merit order is respected

Case d) $c_I = \$30/\text{MWh}$, $c_F = c_F^- = \$35/\text{MWh}$, $c_F^+ = \$40/\text{MWh}$.

Asymmetric balancing costs: upward regulation is more expensive than downward regulation (more typical, perhaps)

$\frac{c_I}{c_F^+} = \frac{30}{40} > \frac{c_F - c_F^-}{c_F^+ - c_F^-} = 0 \Rightarrow$ the merit order is broken

Example: Four cases

	Stochastic				Inefficient			
	$p_I^\star$	$p_F^\star$	$p_W^\star$	cost	$p_I^\star$	$p_F^\star$	$p_W^\star$	cost
Case a)	188	12	50	6140	200	0	50	6198
Case b)	187	0	63	6096	200	0	50	6173
Case c)	187	0	63	6096	200	0	50	6173
Case d)	187	53	10	6096	200	0	50	6198

Table: Comparison of market-clearing outcomes for $\kappa = 0.5$. Dispatch results are given in MWh and costs in \$. System load equal to 250 MWh

Conventional two-stage market with virtual bidding

Clearing of the forward market

$$\underset{p_F, p_I, p_W}{\text{Minimize}} \quad c_F p_F + c_I p_I \tag{11a}$$

$$\text{s.t.} \quad p_F + p_I + p_W + p_V = L : \lambda^f \tag{11b}$$

$$p_W \leqslant \widehat{W} \tag{11c}$$

$$p_F, p_W, p_I \geqslant 0, \tag{11d}$$

Clearing of the balancing market

$$\underset{p_F^+(\omega), p_F^-(\omega), \Delta p^W(\omega)}{\text{Minimize}} \quad p_F^+(\omega) c_F^+ - p_F^-(\omega) c_F^- \tag{12a}$$

$$\text{s.t.} \quad p_F^+(\omega) - p_F^-(\omega) + \Delta p_W(\omega) + \Delta p_V = 0 : \lambda^b(\omega) \tag{12b}$$

$$p_F^-(\omega) \leqslant p_F \tag{12c}$$

$$0 \leqslant p_W + \Delta p_W(\omega) \leqslant W(\omega) \tag{12d}$$

$$p_F^+(\omega), p_F^-(\omega) \geqslant 0, \tag{12e}$$

where p_W and p_F are given by problem (11).

Arbitrager's problem

$$\underset{p_V, \Delta p_V}{\text{Maximize}} \quad p_V \lambda^f + \int_\Omega \Delta p_V \lambda^b(\omega) f(\omega) d\omega \tag{13a}$$

$$\text{s.t.} \quad p_V + \Delta p_V = 0, \tag{13b}$$

- ✓ p_V and Δp_V are the virtual bids in the day-ahead and balancing markets, in that order
- ✓ λ^f and $\lambda^b(\omega)$ are the forward and balancing prices, respectively
- ✓ We assume the arbitrager bids at zero price
- ✓ The arbitrager has *perfect knowledge of the market price distribution* (induced by the uncertain supply)
- ✓ The short-run equilibrium solution can be found by solving the complementarity problem that results from replacing all these optimization problems with their KKT conditions

Conventional two-stage market with virtual bidding

Proposition

Consider the equilibrium problem that results from simultaneously enforcing the optimality conditions of problems (11), (12) and (13), where $c_I < c_F$ and $c_F^+ \geqslant c_F \geqslant c_F^- \geqslant 0$. The equilibrium solution is given by:

$$\begin{cases} p_W + p_V = L, \;\; p_I = 0, \;\; p_F = 0, \;\; \text{if } 0 \leqslant L \leqslant L_2; \\ p_W + p_V = L_2, \;\; p_I = L - L_2, \;\; p_F = 0, \;\; \text{if } L > L_2, \end{cases}$$

in both cases with $p_W \leqslant \widehat{W}$ and $L_2 = F^{-1}\left(\frac{c_I}{c_F^+}\right)$.

Conventional two-stage market with virtual bidding

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Consider the equilibrium problem that results from simultaneously enforcing the optimality conditions of problems (11), (12) and (13), where $c_I < c_F$ and $c_F^+ \geqslant c_F \geqslant c_F^- \geqslant 0$. The equilibrium solution is given by:

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in both cases with $p_W \leqslant \widehat{W}$ and $L_2 = F^{-1}\left(\frac{c_I}{c_F^+}\right)$.

Remark 1: If $L_2 = \frac{c_I}{c_F^+} \leqslant \frac{c_F - c_F^-}{c_F^+ - c_F^-} = L_1$, the merit order is respected and virtual bidding closes the efficiency gap between the conventional and the stochastic two-stage markets.

Warning: This only holds true on the assumption that $\overline{P}_F \geqslant \overline{W}$ and $\overline{P}_I \geqslant L$ **OR** in the case that $c_F = c_F^+ = c_F^-$. That is, there are also cases in which the merit order is preserved, but virtual bidding does not close the efficiency gap.

Conventional two-stage market with virtual bidding

Proposition

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in both cases with $p_W \leqslant \widehat{W}$ and $L_2 = F^{-1}\left(\frac{c_I}{c_F^+}\right)$.

Remark 2: Virtual bidding is unable to close the efficiency gap in those cases where achieving maximum efficiency requires breaking the merit order

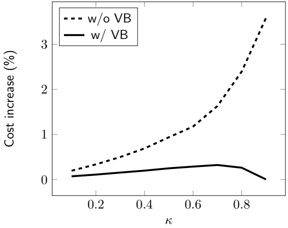
Example: Four cases

	Stochastic				Inefficient w/o VB				Inefficient w/ VB			
	$p_I^\star$	$p_F^\star$	$p_W^\star$	cost	$p_I^\star$	$p_F^\star$	$p_W^\star$	cost	$p_I^\star$	$p_F^\star$	$p_W^\star + p_V^\star$	cost
Case a)	188	12	50	6140	200	0	50	6198	191	0	59	6156
Case b)	187	0	63	6096	200	0	50	6173	187	0	63	6096
Case c)	187	0	63	6096	200	0	50	6173	187	0	63	6096
Case d)	187	53	10	6096	200	0	50	6198	191	0	59	6156

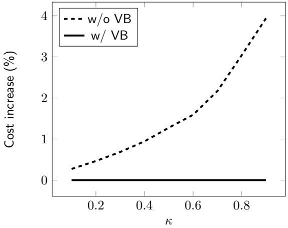
Table: Comparison of market-clearing outcomes for $\kappa = 0.5$. Dispatch results are given in MWh and costs in $\$$. System load equal to 250 MWh

Example: Four cases

Case a)



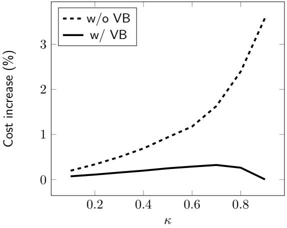
Case c)



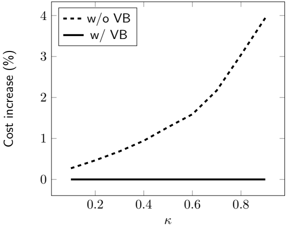
Loss of efficiency (in %) of the conventional two-stage market with and without virtual bidding as a function of the level of wind power penetration

Example: Four cases

Case a)



Case b)



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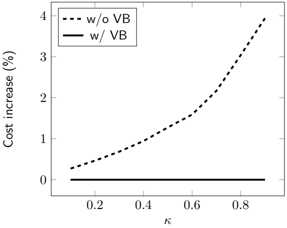
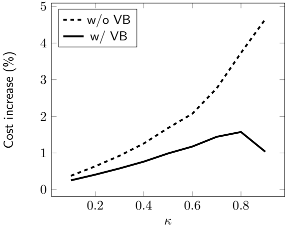


Case d)



Example: Four cases

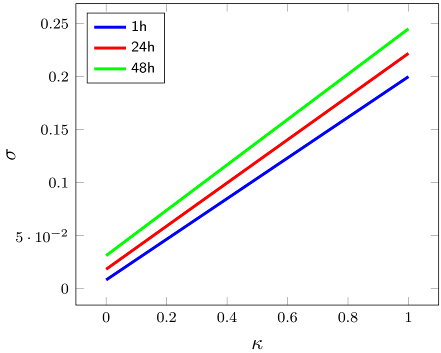
Virtual bidding does *not* take the conventional two-stage market to maximum efficiency in those cases where the optimal dispatch solution breaks the merit order, although it reduces the gap.

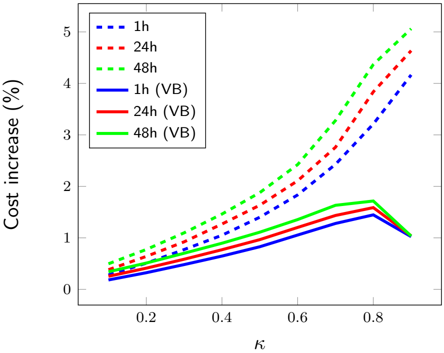
Example: Four cases

Virtual bidding does *not* take the conventional two-stage market to maximum efficiency in those cases where the optimal dispatch solution breaks the merit order, although it reduces the gap.

Remark: In some other cases (not presented here), virtual bidding does *not* close the efficiency gap either even if the merit order is preserved.

Example: Impact of forecast horizon (Case d)





[Fabbri et al., 2005]

Conclusions

- We have identified conditions for market inefficiency in a (stylized) power system that is dispatched by merit order under uncertain supply
- Maximizing market efficiency requires dispatch solutions that the merit order cannot prompt, because they are an intricate function of forward and real-time marginal costs of production and the probabilistic features of the uncertain supply
- (Perfect) virtual bidding can bring the conventional two-stage market closer to maximum efficiency under uncertain supply. We have identified conditions for virtual bidding to close the gap (e.g., if forward and real-time marginal costs of production are equal)
- (Perfect) virtual bidding cannot close the gap in those cases where achieving maximum efficiency requires breaking the merit order. These cases are typical of power systems where providing upward regulation is costly, while the provision of downward regulation entails little or no extra cost at all to the system (appropriate markets for downward operating reserve may be key in these cases)

A. Fabbri, T. Gomez San Roman, J. Rivier Abbad, and V. H. Mendez Quezada. Assessment of the cost associated with wind generation prediction errors in a liberalized electricity market. *IEEE Transactions on Power Systems*, 20(3):1440–1446, 2005.

J. M. Morales, M. Zugno, S. Pineda, and P. Pinson. Electricity market clearing with improved scheduling of stochastic production. *European Journal of Operational Research*, 235(3):765–774, 2014.

Juan M Morales, Antonio J Conejo, Kai Liu, and Jin Zhong. Pricing electricity in pools with wind producers. *IEEE Trans. Power Syst.*, 27(3):1366–1376, 2012.

N. Navid and G. Rosenwald. Market solutions for managing ramp flexibility with high penetration of renewable resource. *IEEE Transactions on Sustainable Energy*, 3(4): 784–790, Oct 2012.

G. Pritchard, G. Zakeri, and A. Philpott. A single-settlement, energy-only electric power market for unpredictable and intermittent participants. *Oper. Res.*, 58(4):1210–1219, Jul./Aug. 2010.

Beibei Wang and Benjamin F. Hobbs. A flexible ramping product: Can it help real-time dispatch markets approach the stochastic dispatch ideal? *Electric Power Systems Research*, 109(0):128 – 140, 2014.

Thanks for your attention!

Website: `https://sites.google.com/site/jnmmgo/`

Example: Impact of forecast horizon (Case d)

$$
\begin{aligned}
z^{\mathrm{S}}(L) &= c_I L - c_F^- \int_0^{L_3} W(\omega) f(\omega) d\omega \\
z^{\mathrm{VB}}(L) &= c_I L - c_F^+ \int_0^{L_2} W(\omega) f(\omega) d\omega
\end{aligned}
$$

where $L_2 = F^{-1}\left(\frac{c_I}{c_F^+}\right) = F^{-1}\left(\frac{3}{4}\right) < F^{-1}\left(\frac{c_I}{c_F^-}\right) = F^{-1}\left(\frac{6}{7}\right) = L_3$

$$
\frac{z^{\mathrm{VB}}(L) - z^{\mathrm{S}}(L)}{z^{\mathrm{S}}(L)} = \frac{c_F^- \int_0^{L_3} W(\omega) f(\omega) d\omega - c_F^+ \int_0^{L_2} W(\omega) f(\omega) d\omega}{c_I L - c_F^- \int_0^{L_3} W(\omega) f(\omega) d\omega}
$$