Scenario reduction for futures market trading in electricity markets

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Introduction

- Power producer (thermal generating units)
- Two main trading floors
  - Pool
  - Derivatives market (forward, futures, options, etc.)
- Two sources of risk
  - Price risk (pool price volatility)
  - Availability risk (unexpected unit failures)
- Risk aversion

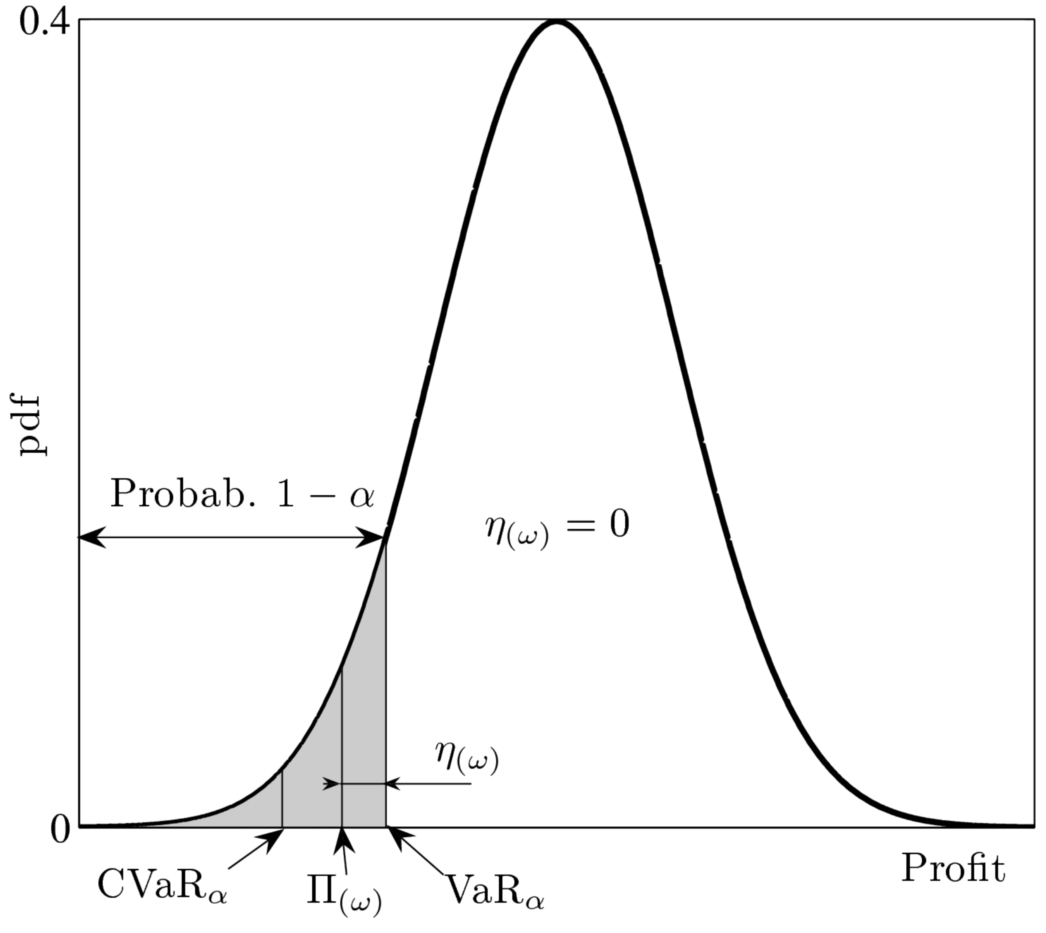
How to make decisions under uncertainty?

Decision Making under Uncertainty

- Stochastic programming (SP)
  - Optimization problems with uncertain parameters
  - “*which would be the best decision?*”
  - Scenarios
    - Possible states of the world
    - Scenario generation procedures
    - Scenario reduction techniques

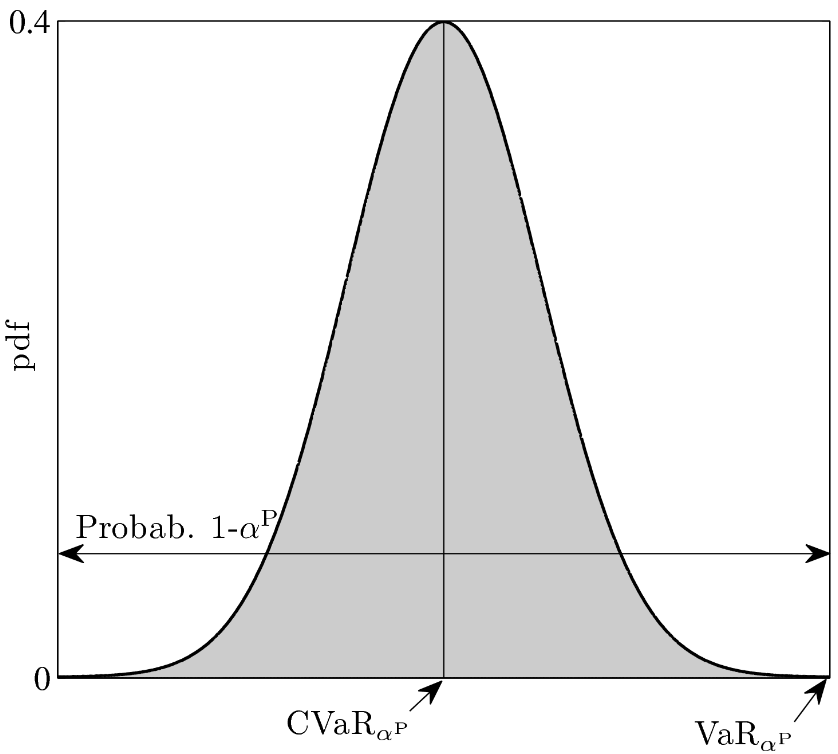
Decision Making under Uncertainty

- Risk management
  - Conditional Value-at-Risk for confidence level α

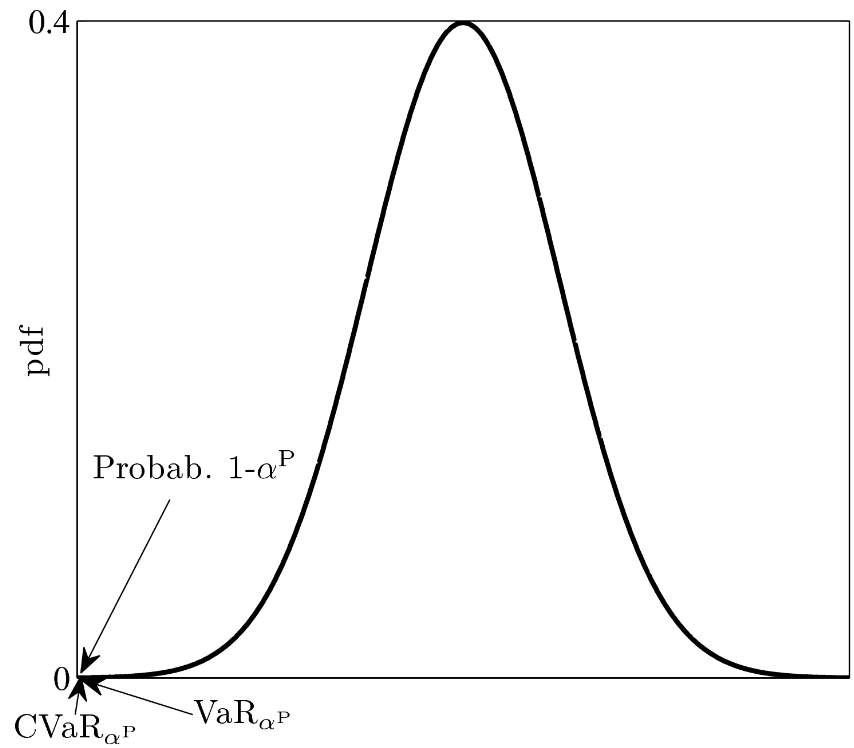


Decision Making under Uncertainty

- Power producer decision model
  - Risk aversion parameter



$\alpha^P = 0$



$\alpha^P \to 1$

Decision Making under Uncertainty

- Two-stage SP with recourse

First-stage or here-and-now decisions x

Realization of stochastic processes $\xi_{(\omega)}$

Second-stage or wait-and-see decisions $y_{(\omega)}$

Decision Making under Uncertainty

- Power producer model

Electricity sold through forward contracts

Realization of pool price and unit availability

Electricity traded in the pool

Decision Making under Uncertainty

- Formulation

Maximize CVaR_{α}(profit)

subject to

Profit of the power producer

Production cost of the units

Technical limits of the units

Energy balance

Arbitrage avoidance

Risk constraints

Binary variable declarations

How to properly represent uncertain parameters?

Scenario Generation and Reduction

- Pool price scenario generation: ARIMA

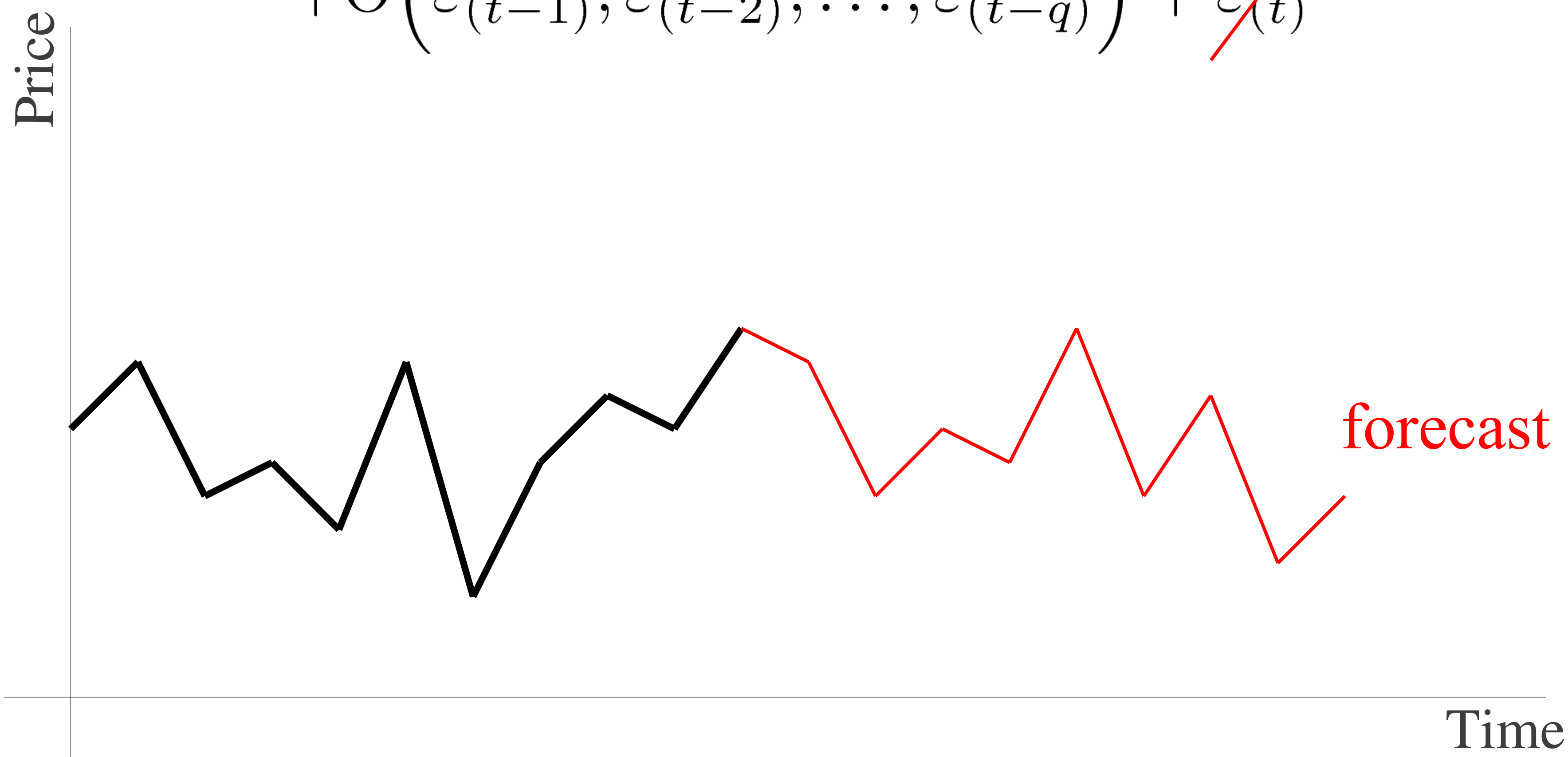
$$\widehat{\lambda^{\mathrm{P}}}_{(t)} = \Phi\left(\lambda^{\mathrm{P}}_{(t-1)}, \lambda^{\mathrm{P}}_{(t-2)}, \ldots, \lambda^{\mathrm{P}}_{(t-p)}\right) +$$

$$+\Theta\left(\varepsilon_{(t-1)}, \varepsilon_{(t-2)}, \ldots, \varepsilon_{(t-q)}\right) + \varepsilon_{(t)}$$

Scenario Generation and Reduction

- Pool price scenario generation: ARIMA

$$\widehat{\lambda^{\mathrm{P}}}_{(t)} = \Phi\left(\lambda^{\mathrm{P}}_{(t-1)}, \lambda^{\mathrm{P}}_{(t-2)}, \ldots, \lambda^{\mathrm{P}}_{(t-p)}\right) + \\ + \Theta\left(\varepsilon_{(t-1)}, \varepsilon_{(t-2)}, \ldots, \varepsilon_{(t-q)}\right) + \cancelto{0}{\varepsilon_{(t)}}$$



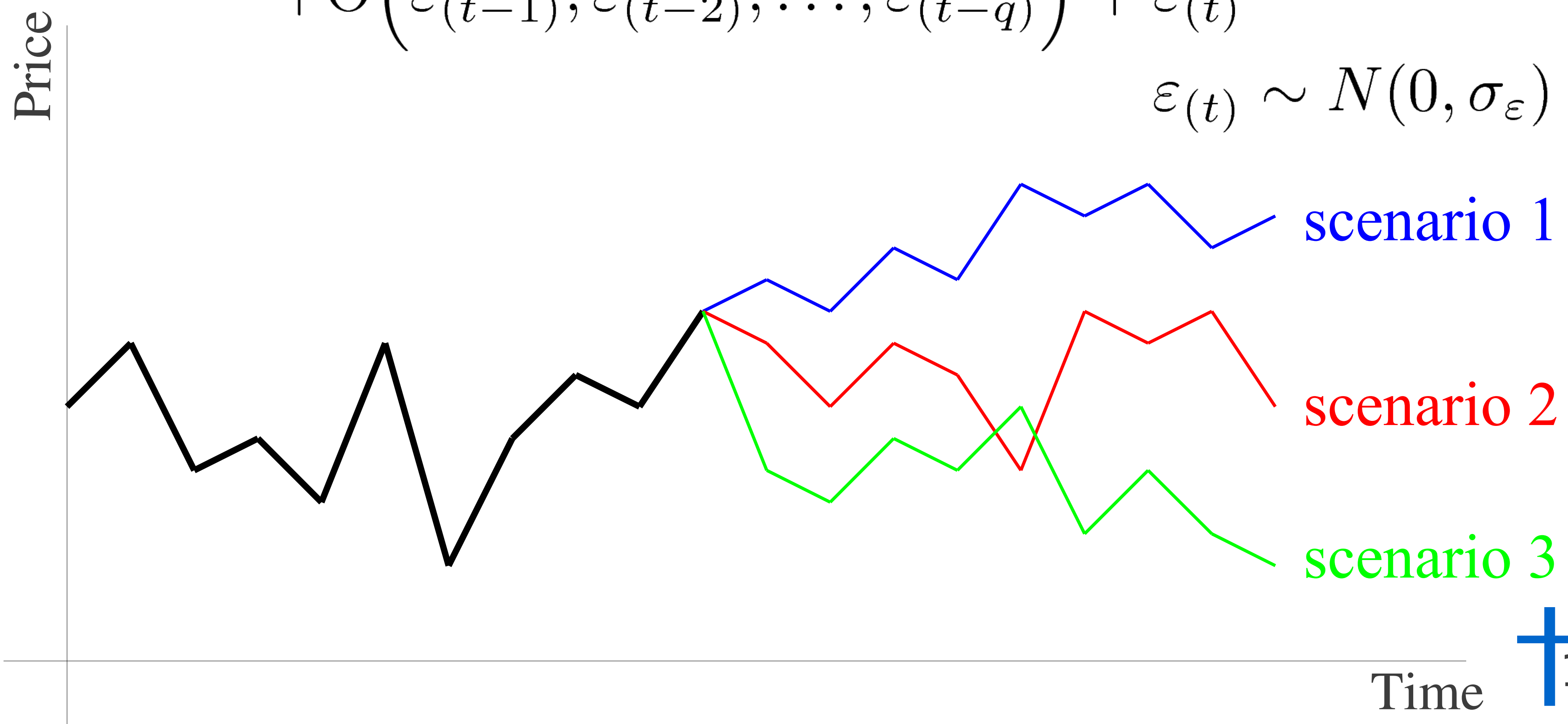
Scenario Generation and Reduction

- Pool price scenario generation: ARIMA

$$\widehat{\lambda^{\mathrm{P}}}_{(t)} = \Phi\left(\lambda^{\mathrm{P}}_{(t-1)}, \lambda^{\mathrm{P}}_{(t-2)}, \ldots, \lambda^{\mathrm{P}}_{(t-p)}\right) +$$

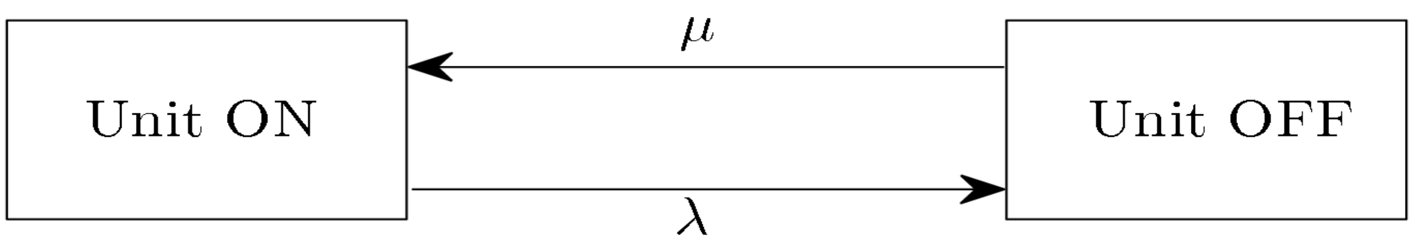
$$+\Theta\left(\varepsilon_{(t-1)}, \varepsilon_{(t-2)}, \ldots, \varepsilon_{(t-q)}\right) + \varepsilon_{(t)}$$

$$\varepsilon_{(t)} \sim N(0, \sigma_{\varepsilon})$$



Scenario Generation and Reduction

- Availability scenario generation



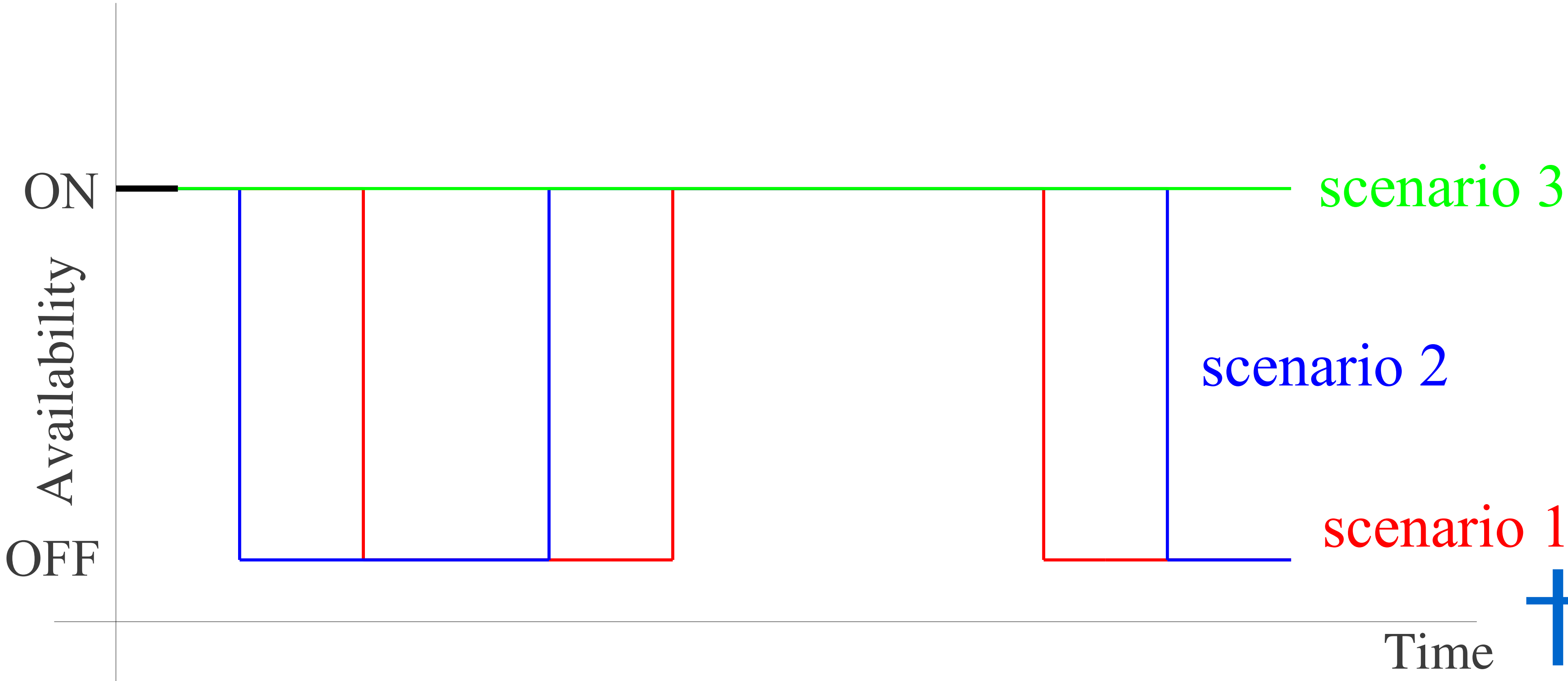
$$p(k_{(t)} = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu \cdot (k_{(t_0)} - 1) + \lambda}{\lambda + \mu} e^{-(\lambda+\mu)(t-t_0)}$$

$$\mu = \frac{1}{\mathrm{MTTR}} \qquad \lambda = \frac{1}{\mathrm{MTTF}} \qquad \mathrm{FOR} = \frac{\mathrm{MTTR}}{\mathrm{MTTF} + \mathrm{MTTR}}$$

Scenario Generation and Reduction

- Availability scenario generation

$$p(k_{(t)} = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu \cdot (k_{(t_0)} - 1) + \lambda}{\lambda + \mu} e^{-(\lambda+\mu)(t-t_0)}$$

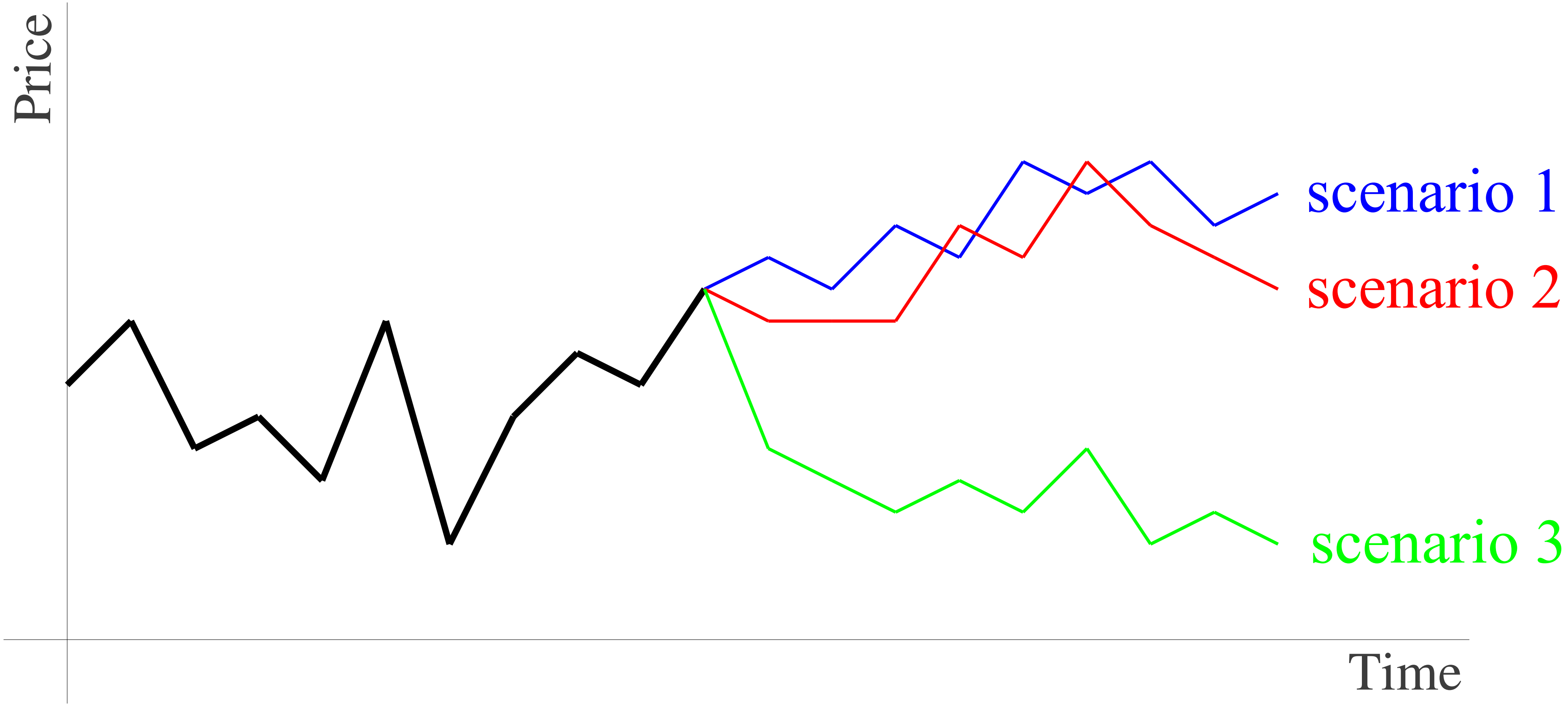


Scenario Generation and Reduction

- Final tree combining all pool price scenarios with all availability scenarios
- The higher the number of scenarios, the better characterized the uncertain parameters are
- Optimization problem may become intractable
- Scenario reduction techniques?

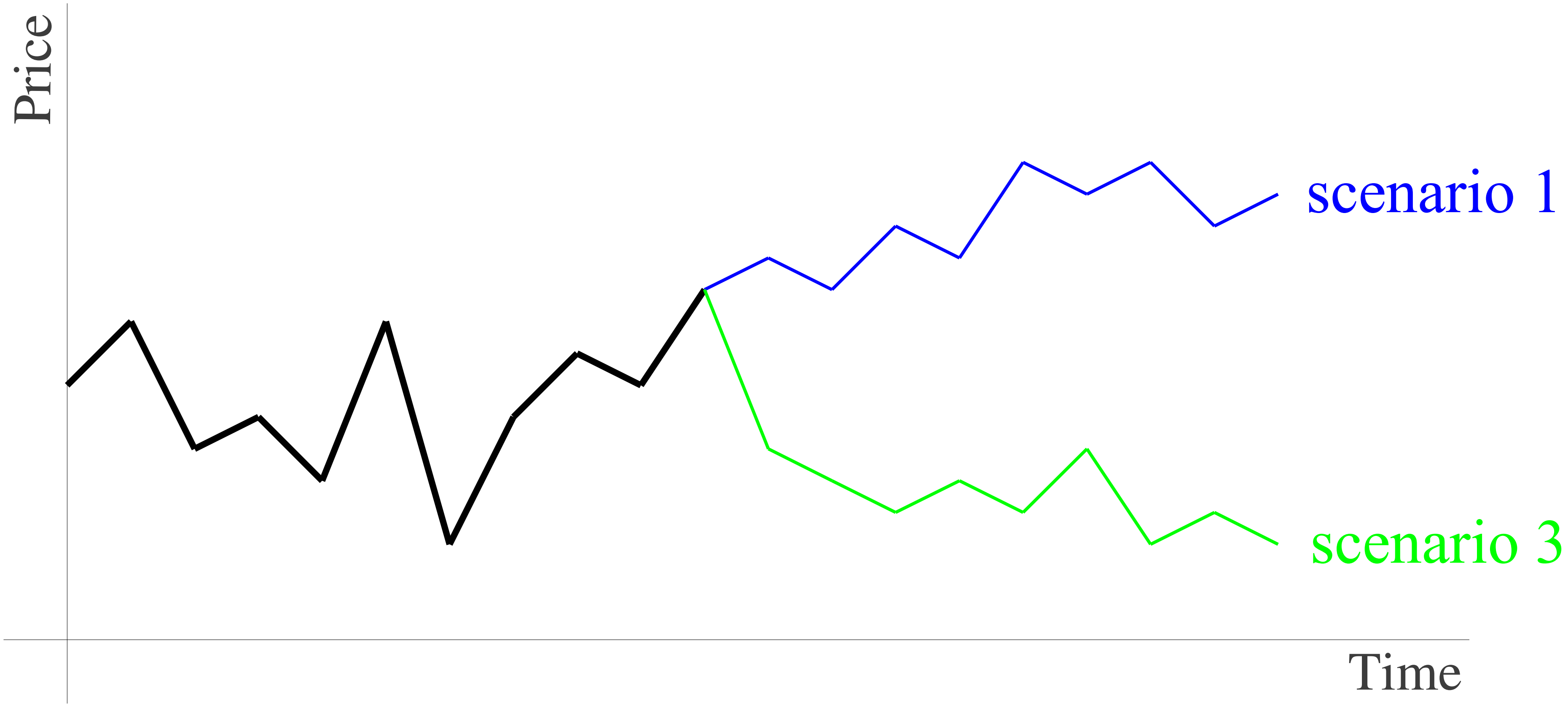
Scenario Generation and Reduction

- Scenario reduction: the idea



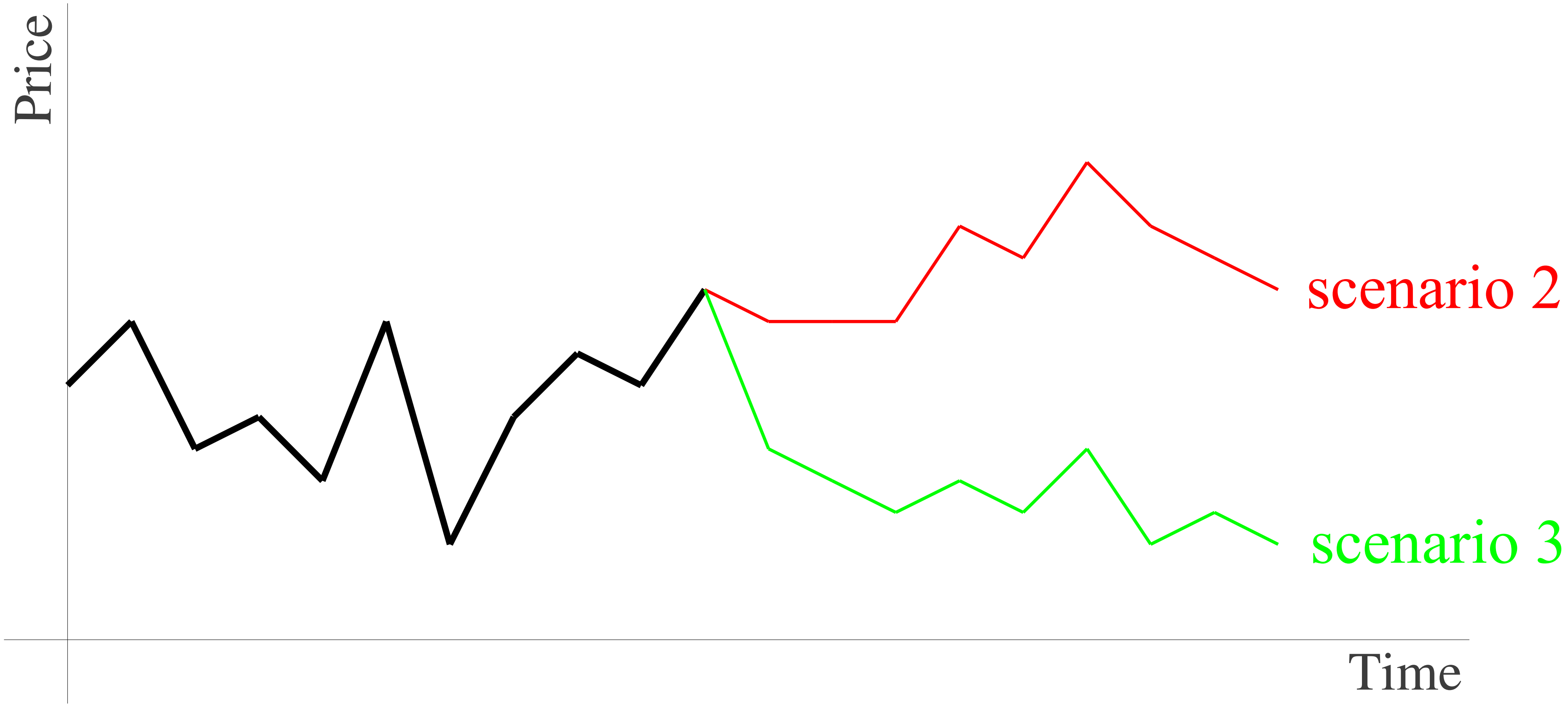
Scenario Generation and Reduction

- Scenario reduction: the idea



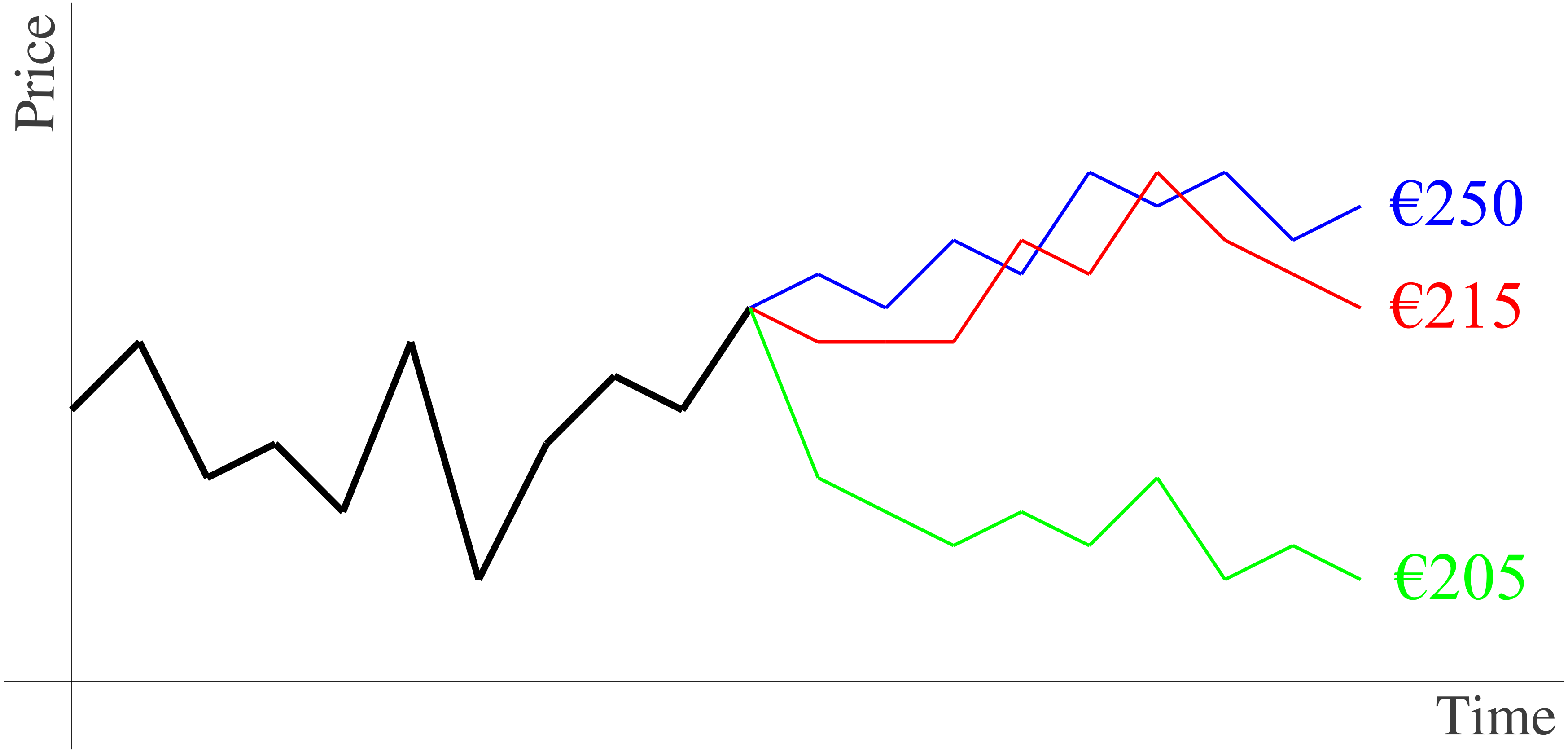
Scenario Generation and Reduction

- Scenario reduction: the idea



Scenario Generation and Reduction

- Scenario reduction: the idea



Scenario Generation and Reduction

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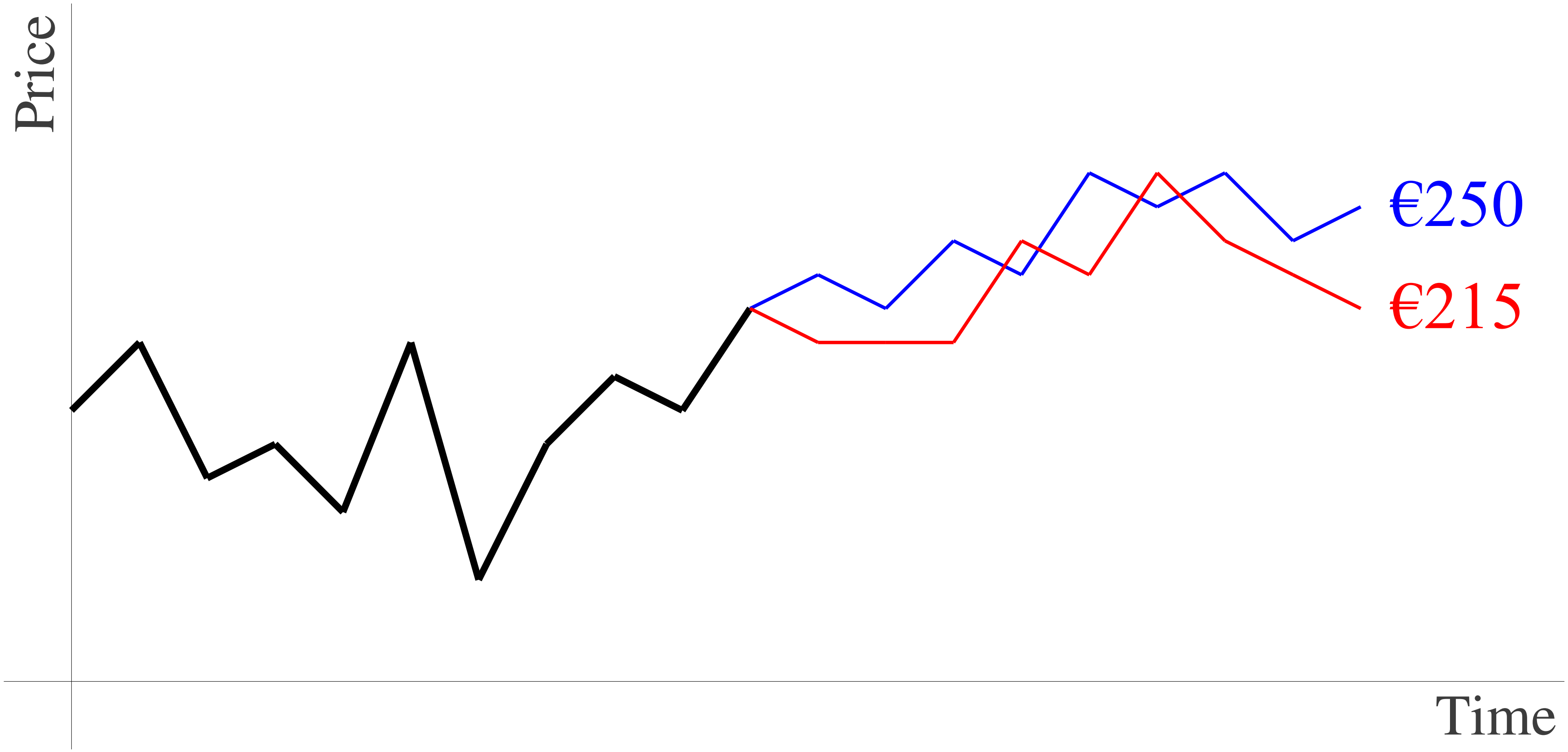

Scenario Generation and Reduction

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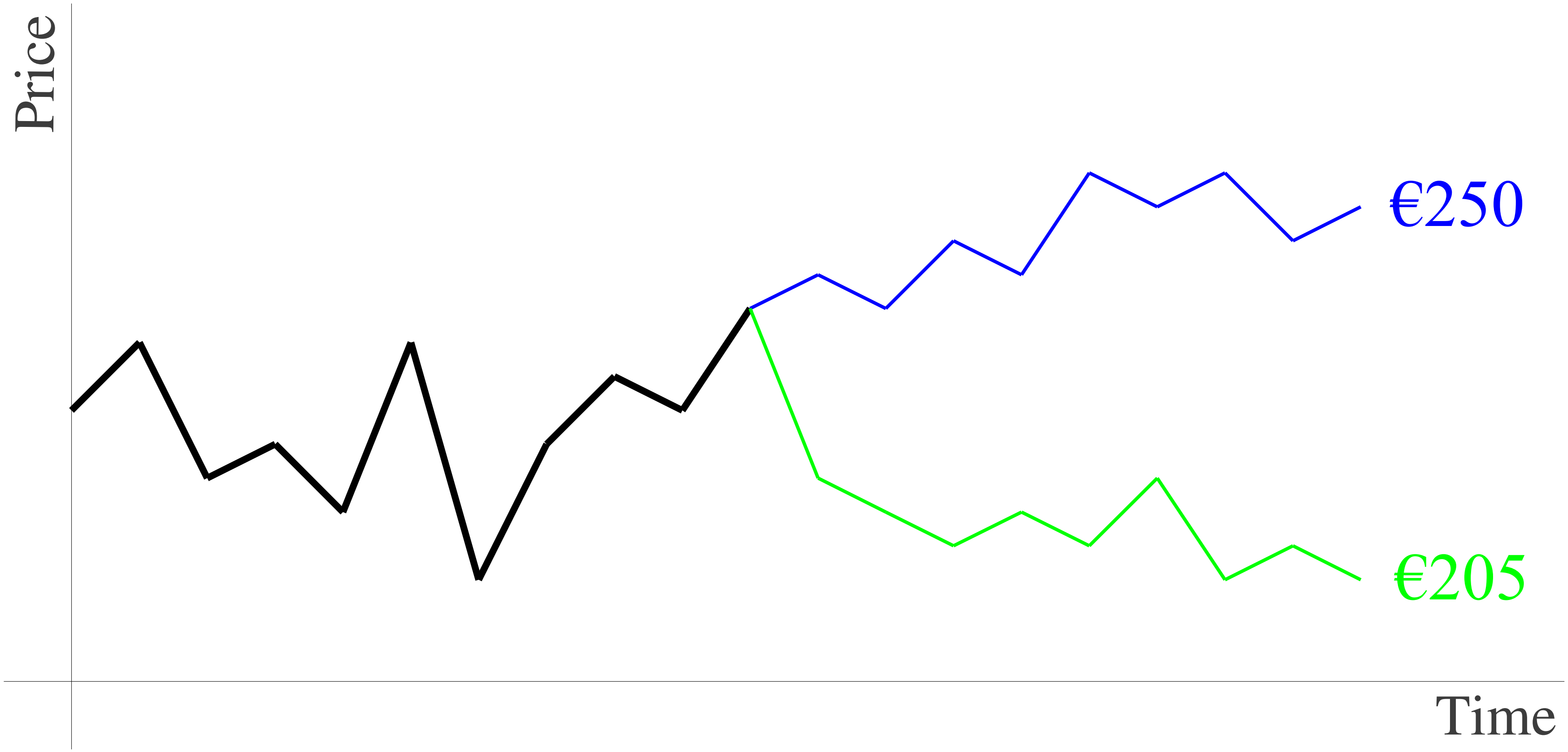


Scenario Generation and Reduction

- Scenario reduction: the theory
  - Kantarovich distance for two-stage SP

$$D_K(Q, Q') = \sum_{\omega \in \Omega \setminus \Omega_S} \pi_\omega \min_{\omega' \in \Omega_S} c(\omega, \omega')$$

Distance between
two scenarios

  - Fast forward selection

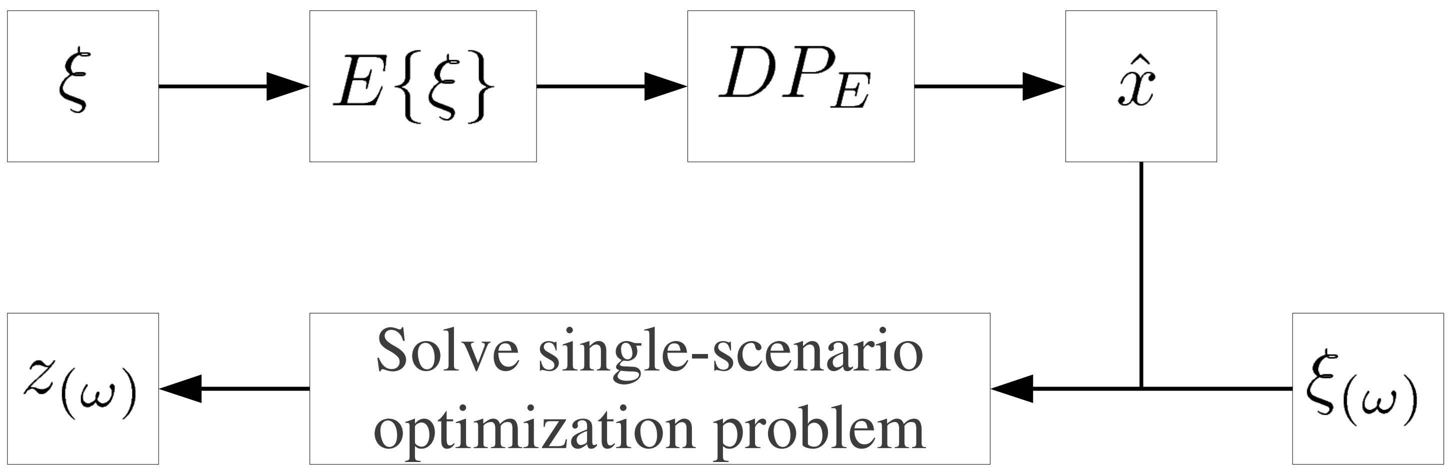
Scenario Generation and Reduction

- Scenario reduction: Technique A

$$c(\omega, \omega') = \|\xi_{(\omega)} - \xi_{(\omega')}\| = \sqrt{\sum_{i=1}^{n} \left(\xi^i_{(\omega)} - \xi^i_{(\omega')}\right)^2}$$

Scenario Generation and Reduction

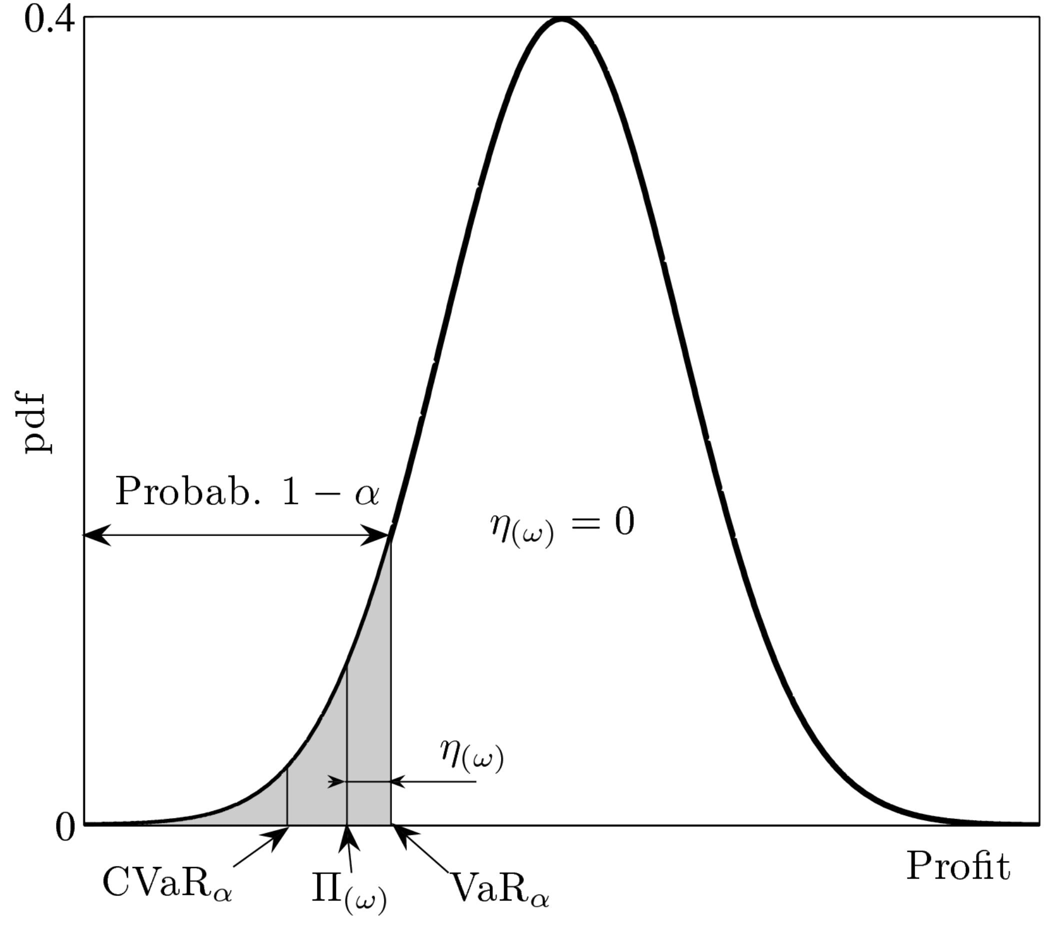
- Scenario reduction: Technique B



$$c(\omega, \omega') = |z_{(\omega)} - z_{(\omega')}|$$

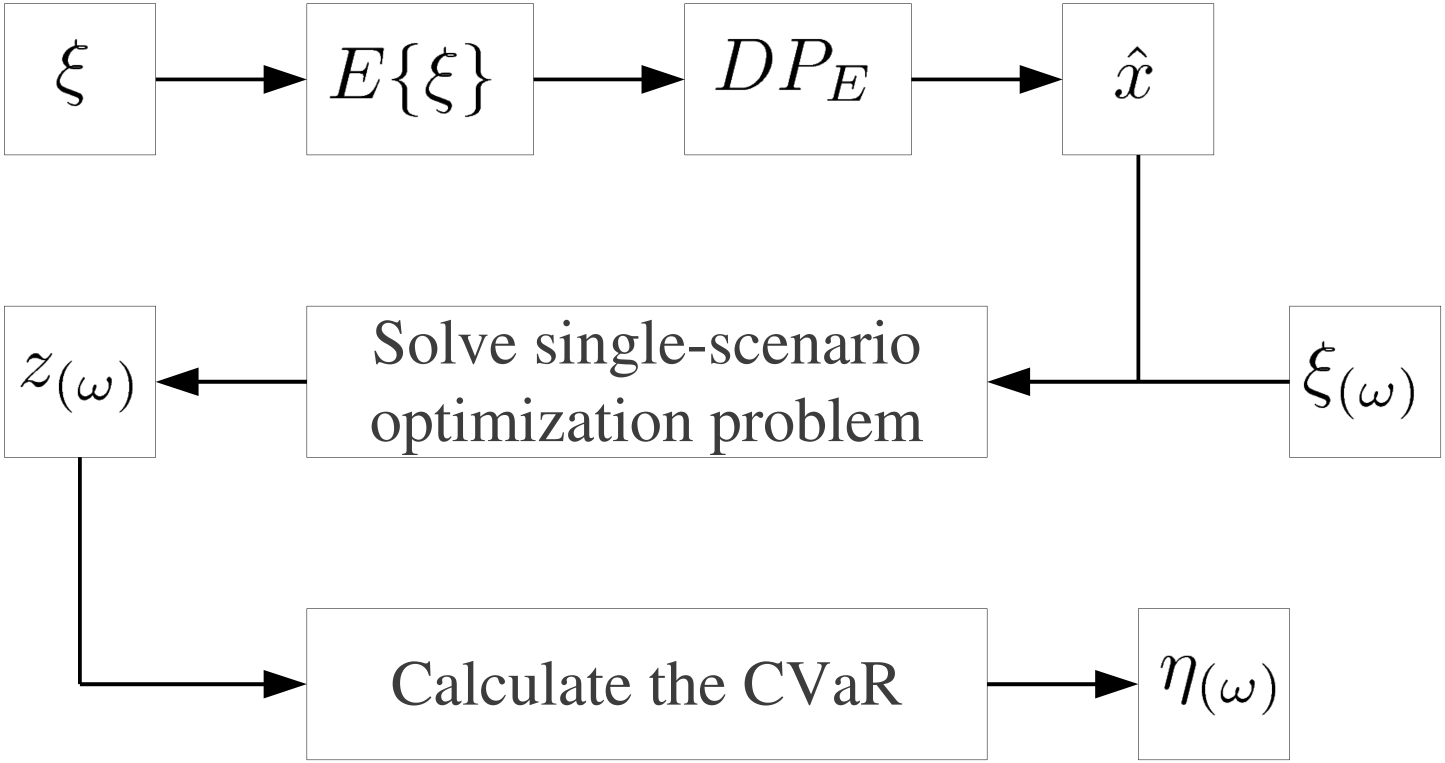
Scenario Generation and Reduction

- Scenario reduction: Technique C



Scenario Generation and Reduction

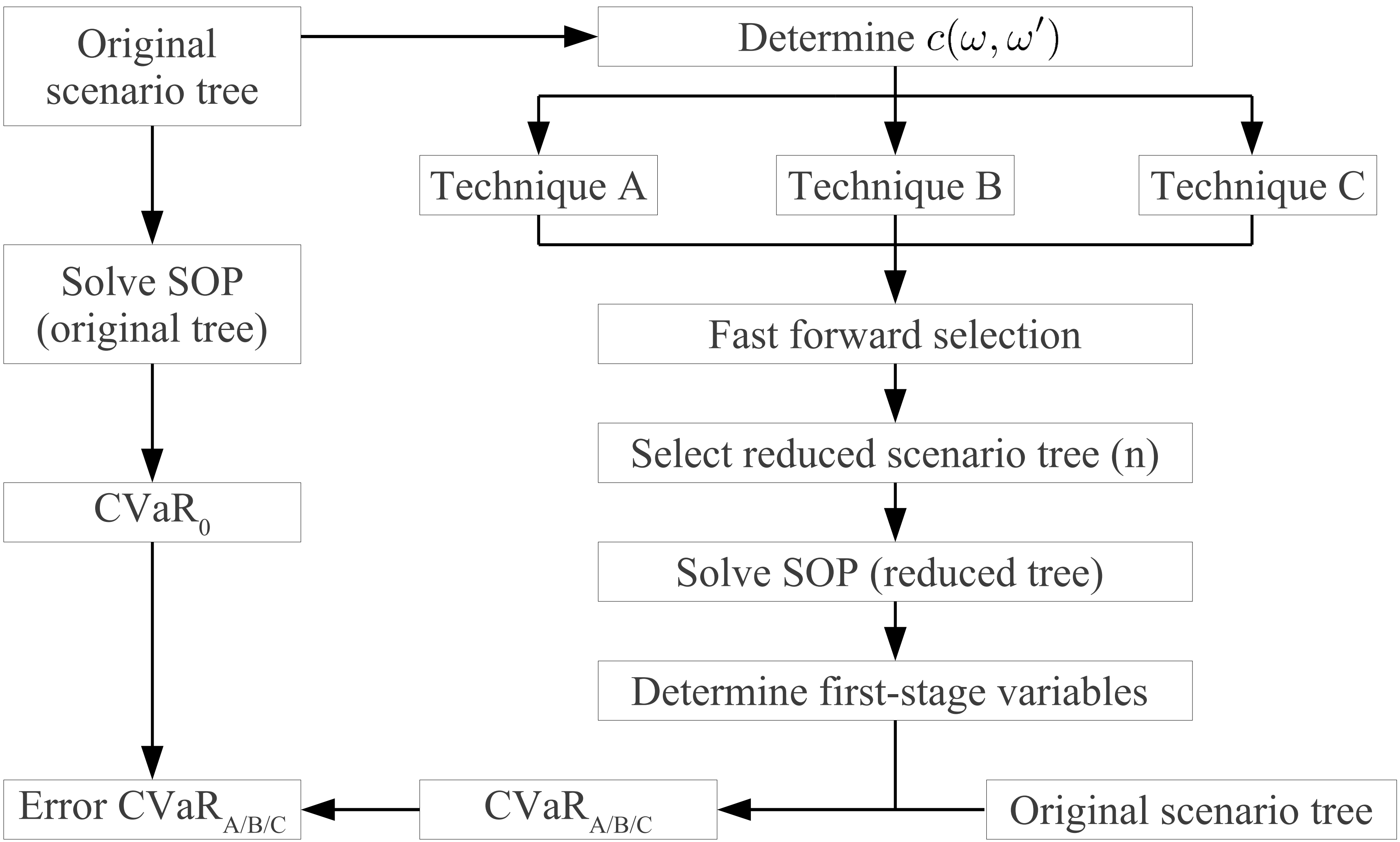
- Scenario reduction: Technique C



$$c(\omega, \omega') = |\eta_{(\omega)} - \eta_{(\omega')}|$$

Scenario Generation and Reduction

- Comparison scenario reduction techniques



Scenario Generation and Reduction

- Case study 1: Reduction of pool price scenarios
  - Study horizon of 1 month

Scenario Generation and Reduction

- Case study 1: Reduction of pool price scenarios
  - Study horizon of 1 month
  - One generating unit
    - ✓ Maximum output 450 MW
    - ✓ Minimum output 50 MW
    - ✓ Piecewise linear production cost
    - ✓ Non-failing unit

Scenario Generation and Reduction

- Case study 1: Reduction of pool price scenarios
  - Study horizon of 1 month
  - One generating unit
  - Pool price uncertainty
    - ✓ ARIMA
    - ✓ Historical data Spanish market year 2000
    - ✓ 200 pool price scenarios

Scenario Generation and Reduction

- Case study 1: Reduction of pool price scenarios
  - Study horizon of 1 month
  - One generating unit
  - Pool price uncertainty
  - Forward contracts
    - ✓ 1 monthly forward contracts
    - ✓ 4 weekly forward contracts
    - ✓ 10 sets of forward prices

Scenario Generation and Reduction

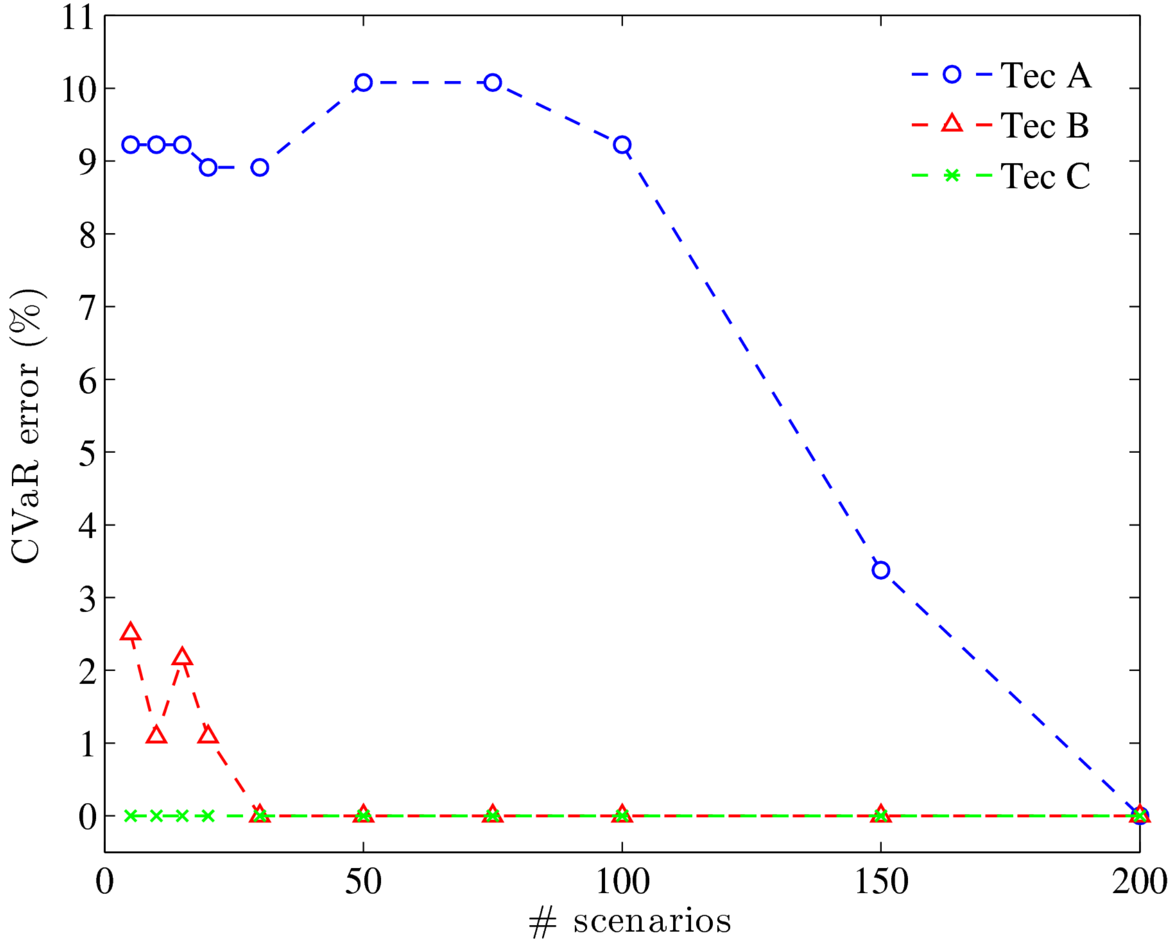
- Case study 1: Reduction of pool price scenarios
  - Study horizon of 1 month
  - One generating unit
  - Pool price uncertainty
  - Forward contracts
  - Risk aversion parameter
    - 6 values: 0, 0.2, 0.4, 0.6, 0.8, 0.9

Scenario Generation and Reduction

- Case study 1: Reduction of pool price scenarios
  - Study horizon of 1 month
  - One generating unit
  - Pool price uncertainty
  - Forward contracts
  - Risk aversion parameter
  - Each technique tested on 60 different problems

Scenario Generation and Reduction

- Case study 1: Reduction of pool price scenarios



Scenario Generation and Reduction

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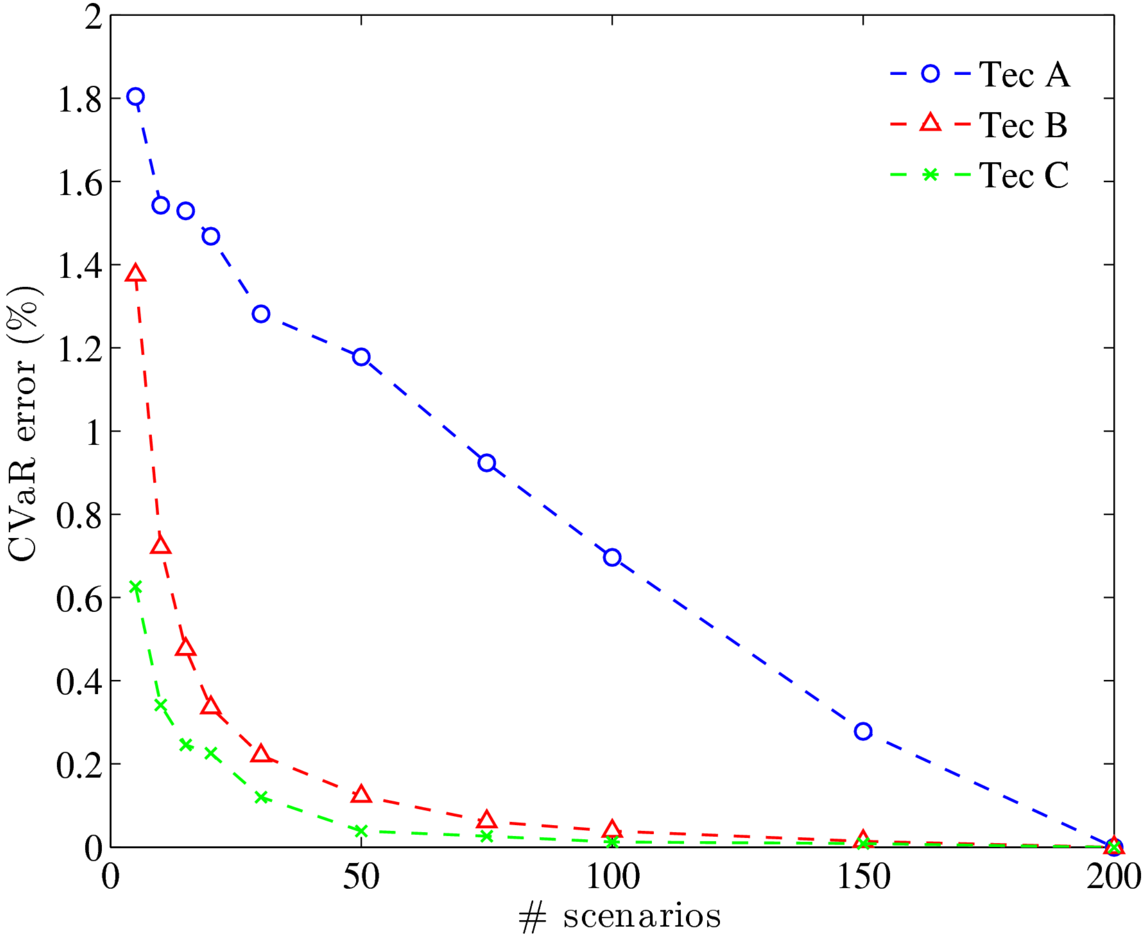

Scenario Generation and Reduction

- Case study 2: Reduction of availability scenarios
  - Study horizon of 1 month

Scenario Generation and Reduction

- Case study 2: Reduction of availability scenarios
  - Study horizon of 1 month
  - One generating unit
    - ✓ Maximum output 450 MW
    - ✓ Minimum output 50 MW
    - ✓ Piecewise linear production cost
    - ✓ Failing unit
      - *FOR = 5%, 10%, 20%*
      - *50 availability scenarios*

Scenario Generation and Reduction

- Case study 2: Reduction of availability scenarios
  - Study horizon of 1 month
  - One generating unit
  - Pool price uncertainty
    - ARIMA
    - Historical data Spanish market year 2000
    - 20 pool price scenarios
    - Aggregated in 3 time steps per day

Scenario Generation and Reduction

- Case study 2: Reduction of availability scenarios
  - Study horizon of 1 month
  - One generating unit
  - Pool price uncertainty
  - Forward contracts
    - ✓ 1 monthly forward contracts
    - ✓ 4 weekly forward contracts
    - ✓ 10 sets of forward prices

Scenario Generation and Reduction

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Scenario Generation and Reduction

- Case study 2: Reduction of availability scenarios
  - Study horizon of 1 month
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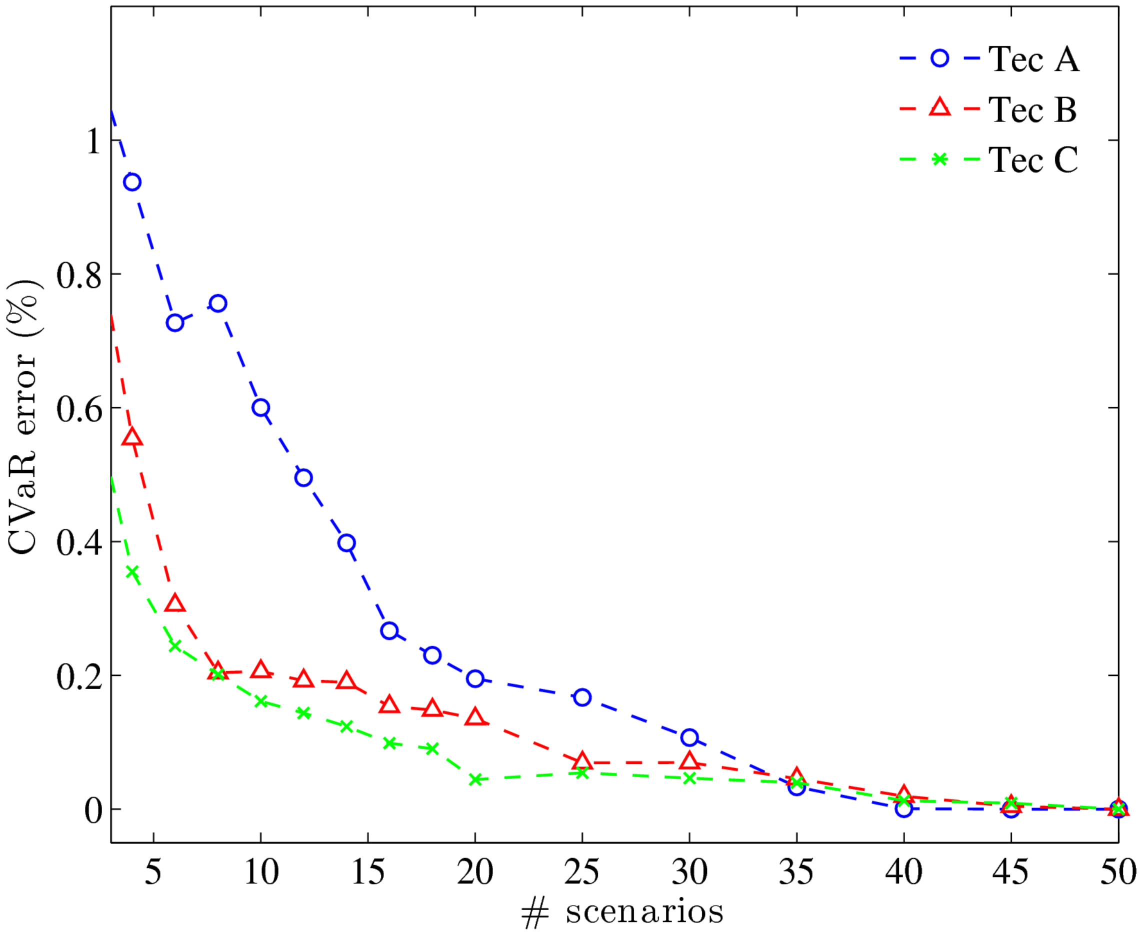
  - Each technique tested on 180 different problems

Scenario Generation and Reduction

- Case study 2: Reduction of availability scenarios



Conclusions

- Scenario reduction
  - A *good* scenario reduction technique is needed to make decisions using stochastic programming
  - The special features of the stochastic model can be accounted for in the scenario reduction technique
  - The risk aversion level of the decision maker can be taken into account in the reduction procedure

Thank you!

Questions?

BACKUP SLIDES

Two-stage SP with recourse

- General form

$$
\begin{aligned}
\underset{x}{\text{Maximize}} \quad & z = c^T x + E[h(x,\omega)] \\
\text{subject to} \quad & Ax \ge b \\
& x \ge 0 \\
\text{where} \quad & h(x,\omega) = \underset{y_{(\omega)}}{\text{Maximize}} \quad g_{(\omega)}^T y_{(\omega)} \\
& \text{subject to} \quad W_{(\omega)} y_{(\omega)} \ge r_{(\omega)} - T_{(\omega)} x, \quad \forall \omega \\
& \qquad\qquad\qquad y_{(\omega)} \ge 0, \quad \forall \omega
\end{aligned}
$$

Two-stage SP with recourse

- Deterministic equivalent program (DEP)

$$
\begin{aligned}
\underset{x,\,y_{(\omega)}}{\text{Maximize}} \quad & z = c^T x + \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} g_{(\omega)}^T y_{(\omega)} \\
\text{subject to} \quad & Ax \geq b \\
& W_{(\omega)} y_{(\omega)} \geq r_{(\omega)} - T_{(\omega)} x, \quad \forall \omega \\
& x \geq 0 \\
& y_{(\omega)} \geq 0, \quad \forall \omega
\end{aligned}
$$

Back

Conditional Value-at-Risk

- Optimization problem

$$\underset{\zeta,\eta_{(\omega)}}{\text{Maximize}} \quad \text{CVaR}_{\alpha}(\Pi) = \zeta - \frac{1}{1-\alpha}\sum_{\omega=1}^{N_{\Omega}} \pi_{(\omega)}\eta_{(\omega)}$$

subject to

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \quad \forall\omega$$

$$0 \leq \eta_{(\omega)}, \quad \forall\omega$$

Back

Maximize CVaR$_\alpha$(profit)

$$\text{Maximize}_{P^{\text{G}}_{(i,\omega,t,b)},P^{\text{P}}_{(\omega,t)},P_{(c)},u_{(i,\omega,t)},\zeta,\eta_{(\omega)}}$$

$$\zeta - \frac{1}{1-\alpha^{\text{P}}} \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \eta_{(\omega)}$$

Back

Profit of the power producer

$$\Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{F}} - C^{\mathrm{G}}_{(\omega)}, \forall \omega$$

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} P^{\mathrm{P}}_{(\omega,t)} L_{(t)}, \forall \omega$$

$$\Pi^{\mathrm{F}} = \sum_{c=1}^{N_C} \lambda_{(c)} P_{(c)} L_{(c)}$$

Back

Production cost of the units

$$C^{\mathrm{G}}_{(i,\omega,t)} = \left(A_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} \lambda_{(i,b)} P^{\mathrm{G}}_{(i,\omega,t,b)} \right) L_{(t)}, \forall i, \forall \omega, \forall t$$

$$C^{\mathrm{G}}_{(\omega)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} C^{\mathrm{G}}_{(i,\omega,t)}, \forall \omega$$

Back

Technical limits of the units

$$u_{(i,\omega,t)}k_{(i,\omega,t)}P_{(i)}^{\mathrm{Min}} \leq P_{(i,\omega,t)}^{\mathrm{G}} \leq u_{(i,\omega,t)}k_{(i,\omega,t)}P_{(i)}^{\mathrm{Max}}, \forall i, \forall \omega, \forall t$$

$$P_{(i,\omega,t)}^{\mathrm{G}} = P_{(i)}^{\mathrm{Min}}u_{(i,\omega,t)} + \sum_{b=1}^{N_B} P_{(i,\omega,t,b)}^{\mathrm{G}}, \forall i, \forall \omega, \forall t$$

$$0 \leq P_{(i,\omega,t,b_1)}^{\mathrm{G}} \leq P_{(i,b_1)}^{\mathrm{Max}} - P_{(i)}^{\mathrm{Min}}, \forall i, \forall \omega, \forall t$$

$$0 \leq P_{(i,\omega,t,b)}^{\mathrm{G}} \leq P_{(i,b)}^{\mathrm{Max}} - P_{(i,b-1)}^{\mathrm{Max}}, \forall i, \forall \omega, \forall t, \forall b = b_2, \ldots, N_B$$

Back

Energy balance

$$\sum_{i=1}^{N_I} P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{P}}_{(\omega,t)} + \sum_{c \in F_{(t)}} P_{(c)}, \forall \omega, \forall t$$

Back

Arbitrage avoidance

$$\sum_{i=1}^{N_I}(k_{(i,\omega,t)} - 1)P_{(i)}^{\mathrm{Max}} \leq P_{(\omega,t)}^{\mathrm{P}}, \forall\omega, \forall t$$

$$0 \leq P_{(c)}, \forall c$$

Back

Risk constraints

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \forall \omega$$

$$0 \leq \eta_{(\omega)}, \forall \omega,$$

Back

Binary variable declarations

$$u_{(i,\omega,t)} \in \{0,1\}, \forall i, \forall \omega, \forall t$$

Back